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Gaussian Process Regression Surrogate Modeling with Transfer Learning for Low Computational Cost Structural Reliability Analysis

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【Background】Necessity to consider uncertainties in infrastructures

- Infrastructures such as bridges are designed for load and strength.
- However, structures may deteriorate and suffer damage, or collapse due to earthquakes or other damage, during the service life of a that.
- This is due to the difference between design and reality.
There are many uncertainties in reality.
- Therefore, a reliability analysis is needed that considers uncertainties related to loads and structural strength.

【Deterioration and damage of bridges】

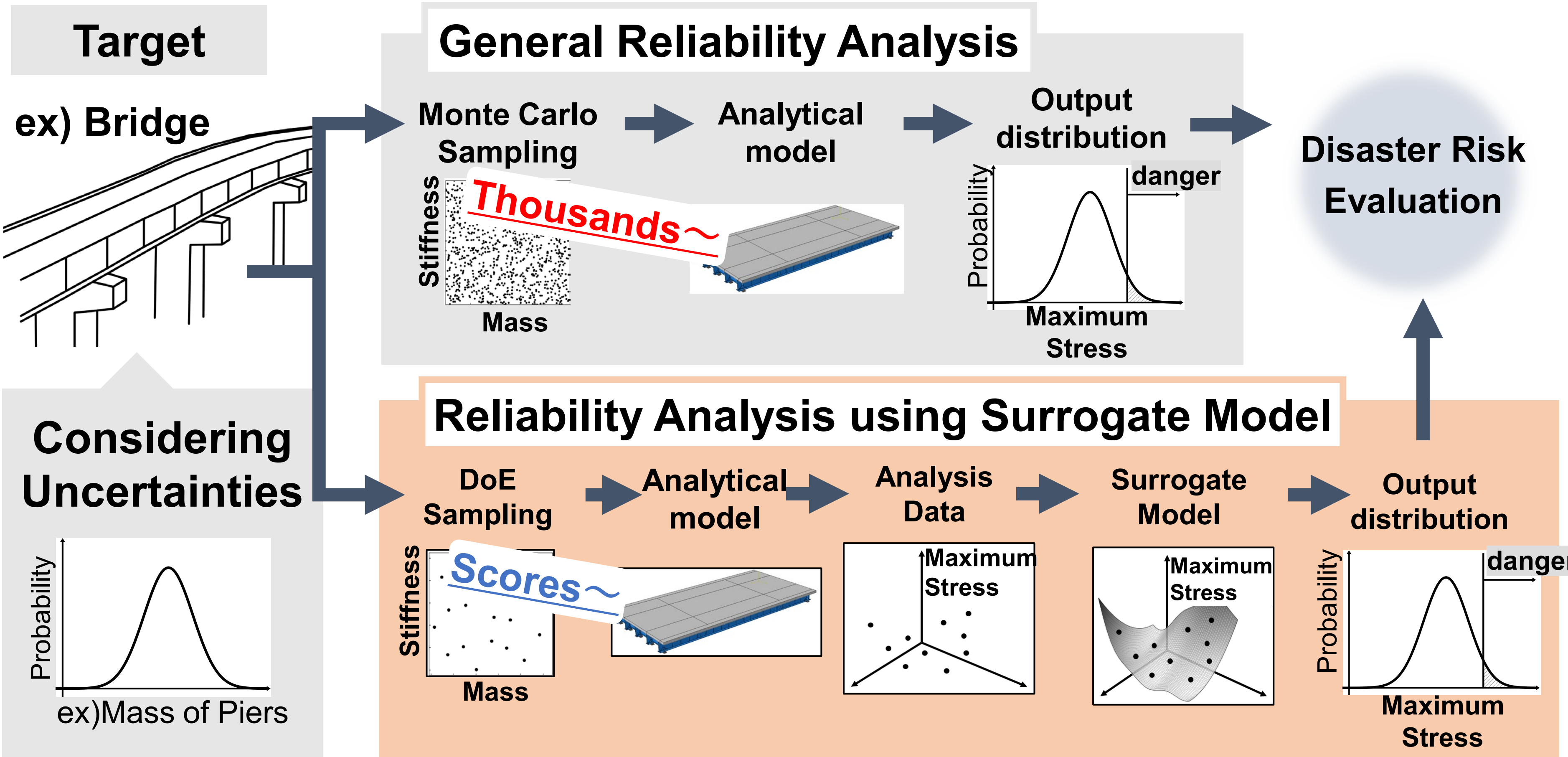
【Examples of serious damage】

Steel pile pier corrosion

(MLIT, Measures to prevent roads from aging, Aging Status)
(MLIT, Anti-aging Initiatives)

(JSCE, Steel Structure Committee)

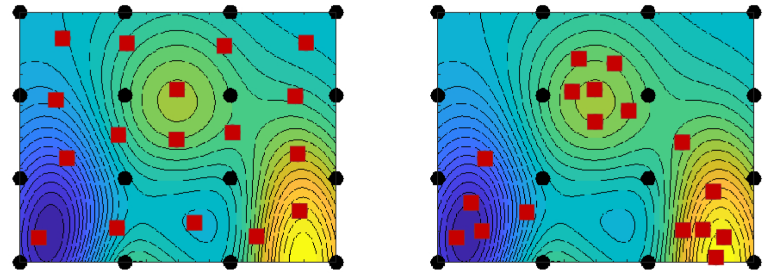
【Background】Reliability Analysis Flow



Surrogate models can reduce computational cost of reliability analysis

【Previous Research】Reduced computational cost of building surrogate models

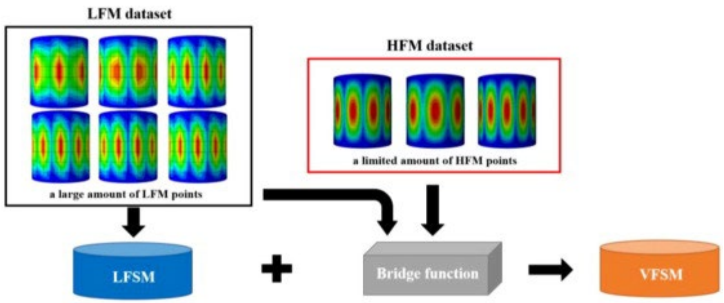
- <u>Adaptive Sampling</u>
  Reduces computational cost by focusing on hard-to-predict points and points of high importance when sampling input parameters
  Echard et al., Structural Safety, 2011



(a) Space-filling design **(b)** Adaptive design

(Jan et al., Archives of Computational Methods in Engineering, 2021)

- <u>Variable fidelity surrogate model</u>
  The use of low-fidelity analysis results with low computational cost reduces the number of targeted high-computational-cost analyses
  Skandalos et al., Structural Safety, 2022



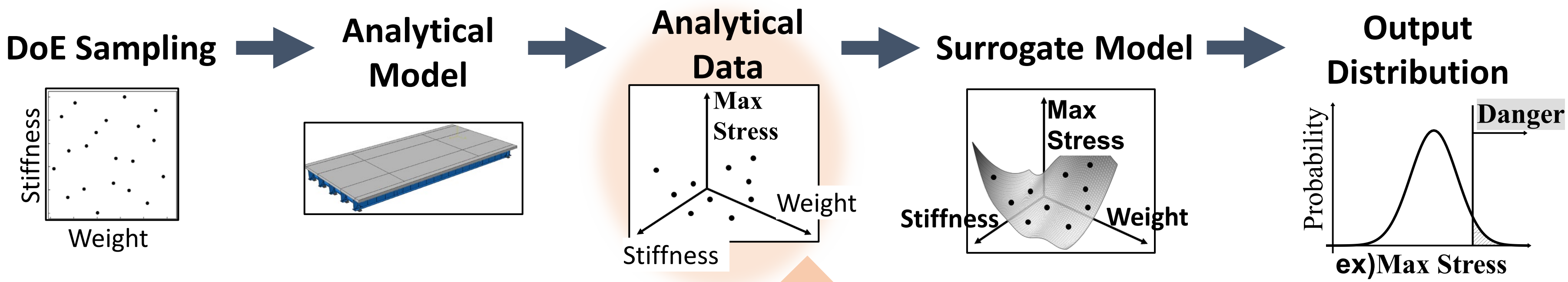
(a) VFSM

(Tian et al., Composite Structures, 2021)

Problem	The surrogate model is valid only for the analysis of the target

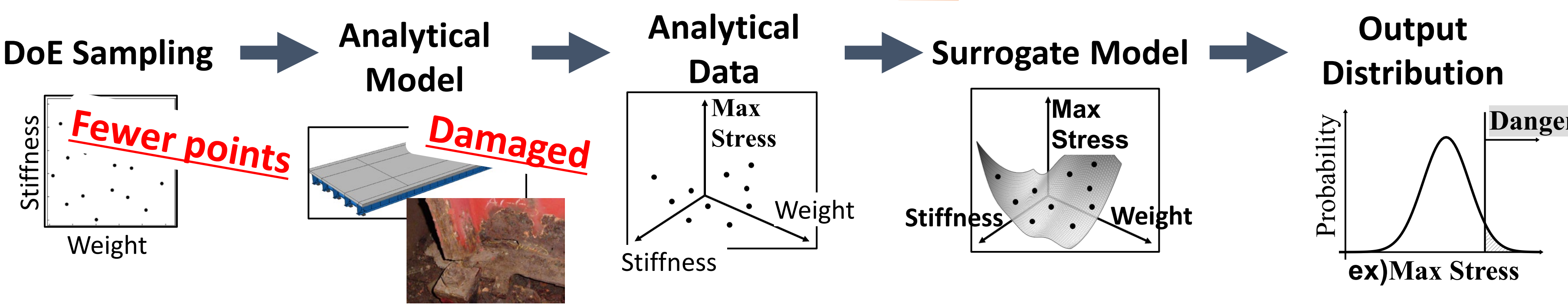
Transfer Learning Gaussian Process Regression Surrogate Model (TL-GPRSM)

When designing a bridge



Transfer learning leverages design-time data

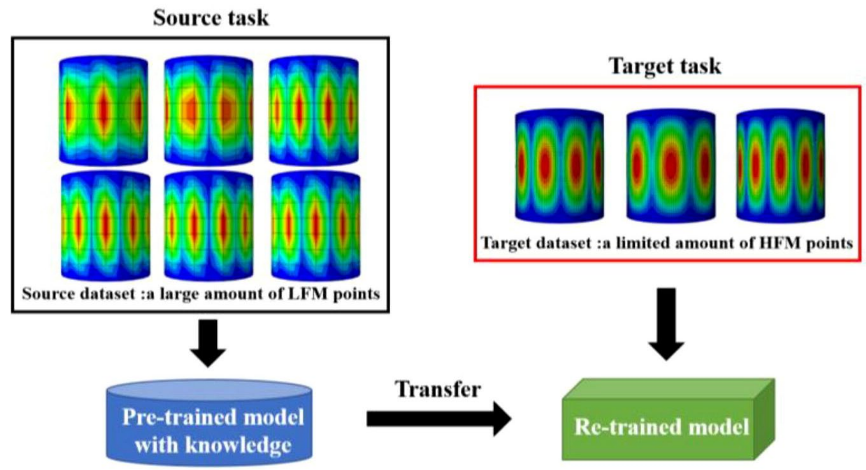
When evaluation of existing bridges



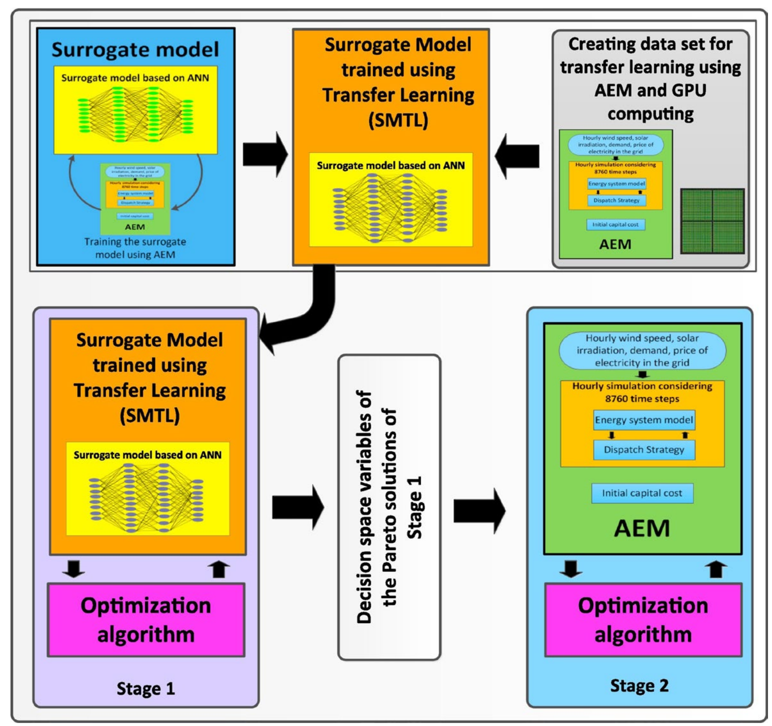
Reduced computational cost for reliability assessment of existing bridges by building a TL-GPRSM using transfer learning

【Previous Reserch】Surrogate model with transfer learning

- Application of Transfer Learning to Variable Fidelity Surrogate Models：
  Transfer learning of DNN models trained on low-fidelity data to high-fidelity domains
  (Tian et al., Composite Structures, 2021)



- Surrogate models for energy system optimization：
  Using transfer learning to respond to environmental changes such as wind and solar
  (Perera et al., Applied Energy, 2019)



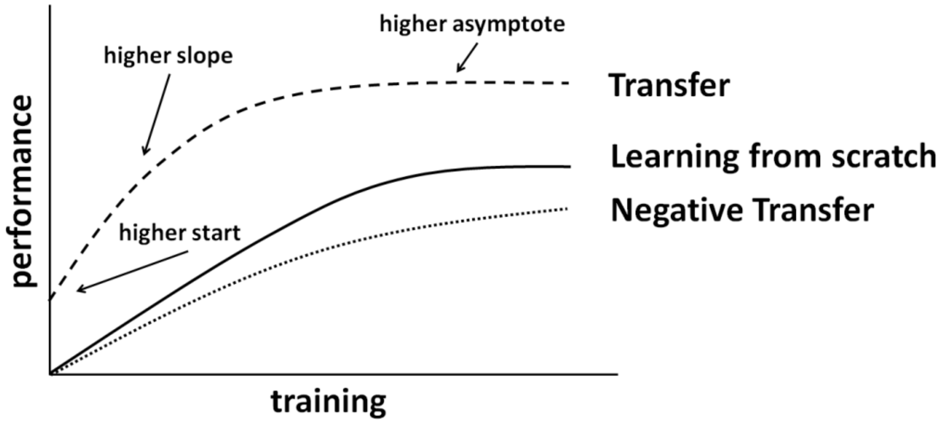
The case of unsuccessful transfer learning is not anticipated.

Issues in transfer learning

- Negative Transfer
  Transfer learning degrades the performance of machine learning models.

⇩

The cause is low similarity between the source and destination data.



Tommasi et al., IEEE transactions on pattern analysis and machine intelligence, 2013

The possibility of negative transfer should be considered

Gaussian Process Regression (GPR) with ARD Kernel

GPR

- **Nonparametric**
- **Non-linear regression**

$$y = f(\mathbf{x})$$

$$f \sim GP(\mathbf{0}, k(\mathbf{x}, \mathbf{x}'))$$

$$\mathbf{y} \sim \mathcal{N}(0, \mathbf{K})$$

$\mathbf{x}$: input vector

$\mathbf{y}$: output vector

k : kernel function

$\mathbf{K}$: kernel matrix

Kernel Matrix

$$K_{\mathbf{nm}} = k(\mathbf{x}_{\mathbf{n}}, \mathbf{x}_{\mathbf{m}})$$

K_{nm} : elements of kernel matrix

ARD Kernel Function

ARD：Automatic Relevance Determination

$$k(\mathbf{r}) = \sigma\left(1 + \sqrt{5}\sum_{i=1}^{D}\frac{r_i}{l_i} + \frac{5}{3}\sum_{i=1}^{D}\frac{r_i^2}{l_i^2}\right)\exp\left(-\sqrt{5}\sum_{i=1}^{D}\frac{r_i}{l_i}\right)$$

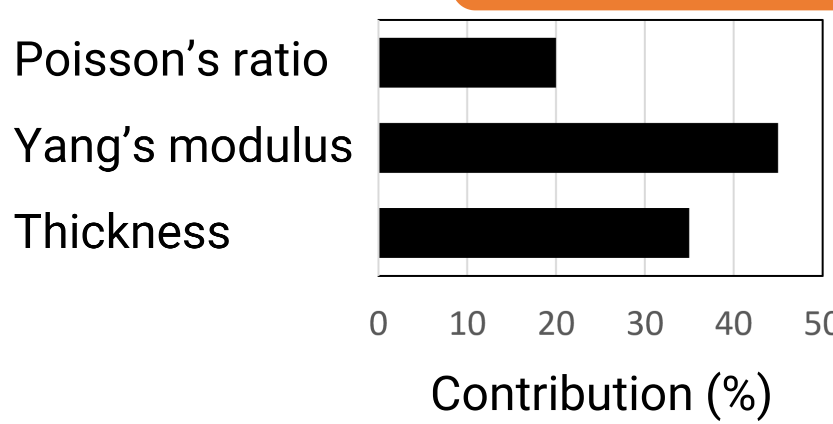
Matern5/2 kernel

Length Scale (l_i)

Represents the contribution of each input variable to the output

ARD Kernel

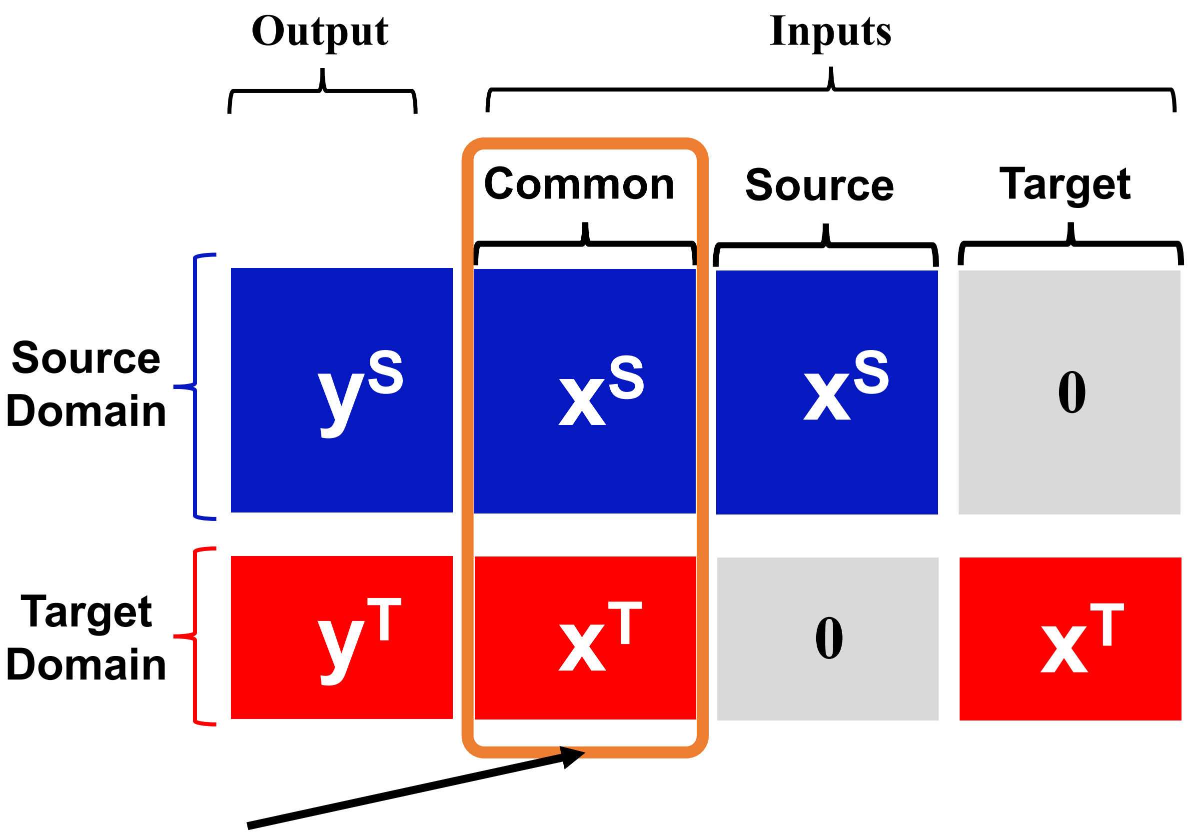
Estimate the contribution of input parameters

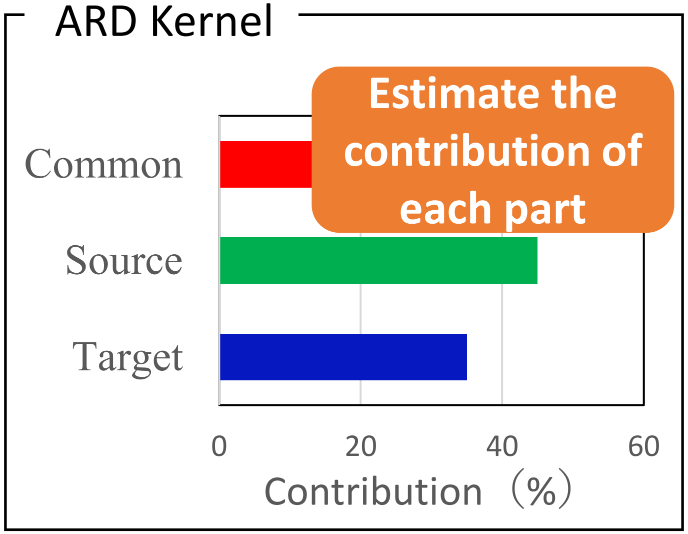


Transfer Learning in Gaussian Process Regression

$$\Phi^{s}(x) = \langle \Phi(x), \Phi(x), \mathbf{0} \rangle$$

$$\Phi^{t}(x) = \langle \Phi(x), \mathbf{0}, \Phi(x) \rangle$$





The greater the contribution of the Common part, the greater the effect of transfer learning.

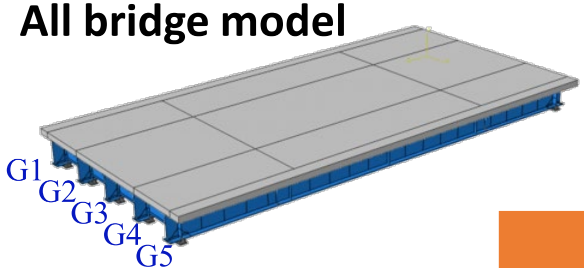
H. Daumé III, Frustratingly Easy Domain Adaptation, ArXiv [Cs.LG]. (2009). http://arxiv.org/abs/0907.1815.

FE model of bridge

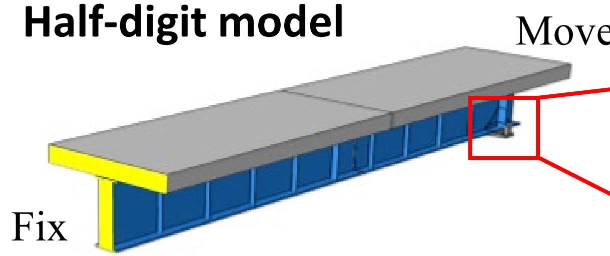
Analysis model: Standard simple I-girder bridge

Length	: 20000 mm	Steel Wire Bearing	: Solid Elements
Width	: 10700 mm	Number of elements	: 104799
Girder	: Shell element	Analysis software	: Abaqus
Floor slab	: Shell element		

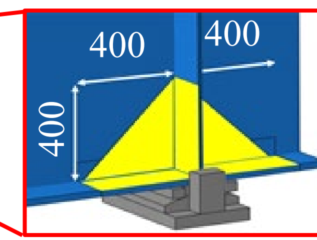
All bridge model



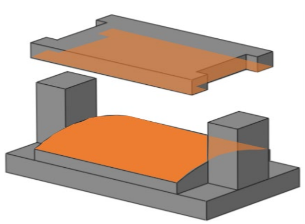
Half-digit model



Corrosive area



Friction considerations



Adjustment to match behavior with all bridge model

Uncertainty setting

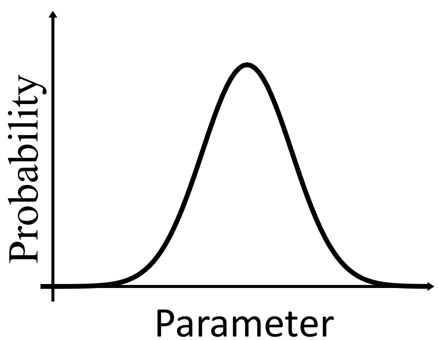
FE Model Parameters (Units)			At Design		At Damage	
			Nominal	COV	Nominal	COV
#1	D_c	Density of concrete slab(kg/m^3)	2400	0.0171	*	*
#2	E_s	Young's modulus of steel main girders(GPa)	200	0.0450	*	*
#3	**E_c**	**Young's modulus of concrete slab(GPa)**	**25**	**0.0167**	**22.5**	**0.0333**
#4	E_b	Young's modulus of steel bearings(GPa)	200	0.0450	*	*
#5	V_s	Poisson's ratio of steel main girder	0.3	0.0910	*	*
#6	V_c	Poisson's ratio of concrete slab	0.2	0.0167	*	*
#7	V_b	Poisson's ratio of steel bearing	0.3	0.0910	*	*
#8	**C_f**	**Friction coefficient of steel bearing**	**0.2**	**0.0167**	**0.9**	**0.0333**
#9	T_{uf1}	Thickness of upper flange of steel girder at near-end section (mm)	0.0190	0.0121	*	*
#10	T_{uf2}	Thickness of upper flange of steel girder at mid-span section (mm)	0.0300	0.0121	*	*
#11	T_w	Thickness of web plate of steel girder (mm)	0.0090	0.0121	*	*
#12	T_{bf1}	Thickness of lower flange of steel girder at near-ends section (mm)	0.0270	0.0121	*	*
#13	T_{bf2}	Thickness of lower flange of steel girder at mid-span section (mm)	0.0300	0.0121	*	*
#14	T_{stc}	Thickness of stiffener of steel girder at near-ends section (mm)	0.0130	0.0121	*	*
#15	T_{stm}	Thickness of stiffener of steel girder at mid-span section (mm)	0.0100	0.0121	*	*
#16	T_{stn}	Thickness of stiffener of steel girder at other section (mm)	0.0065	0.0121	*	*
#17	**T_{bf-d}**	**Thickness of corroded area in lower flange of steel girder at near-end section (mm)**	**–**	**–**	**0.025**	**0.0270**
#18	**T_{w-d}**	**Thickness of corroded area in web plate of steel girder (mm)**	**–**	**–**	**0.008**	**0.0162**
#19	**T_{st-d}**	**Thickness of corroded area in stiffener of steel girder at near-end section (mm)**	**–**	**–**	**0.012**	**0.0162**

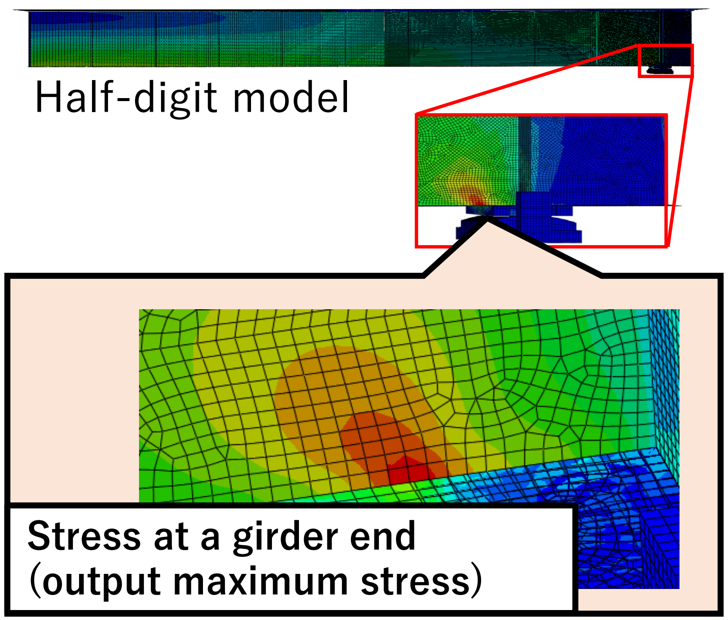
※Determined with reference to previous studies

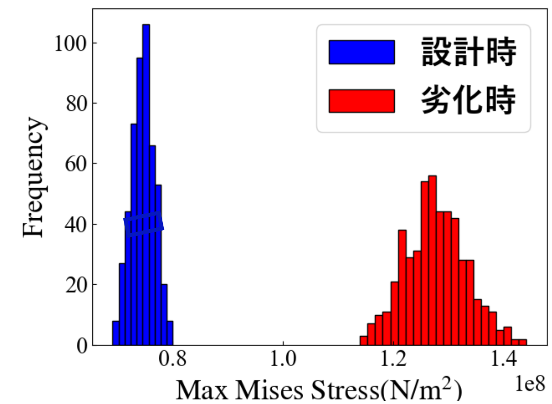
Reliability Analysis Overview

Uncertain Parameters → **FE Analysis (design live load is applied)** → **Maximum Mises Stress Distribution (500 data each)**

At design time
16 variables
At deterioration
19 variables



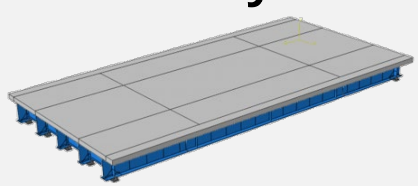




Reliability Analysis Inputs and Outputs

Inputs

Uncertain Parameters → **FE Analysis** → **Output**



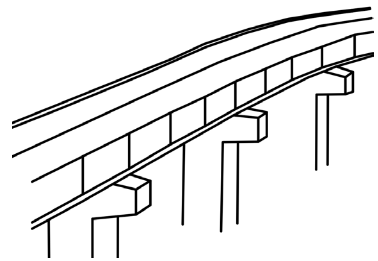
Output

Maximum Mises stress at girder ends

Datas for Transfer Learning

Source Data: At design

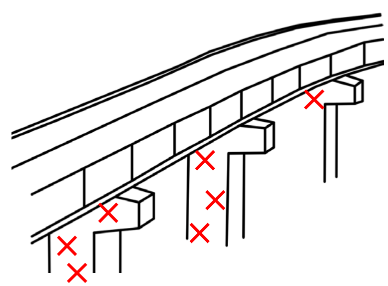
Maximum Stress	Uncertain Parameter
y^S	x^S



Transfer Learning

Target Data: At damage

Maximum Stress	Uncertain Parameter
y^T	x^T



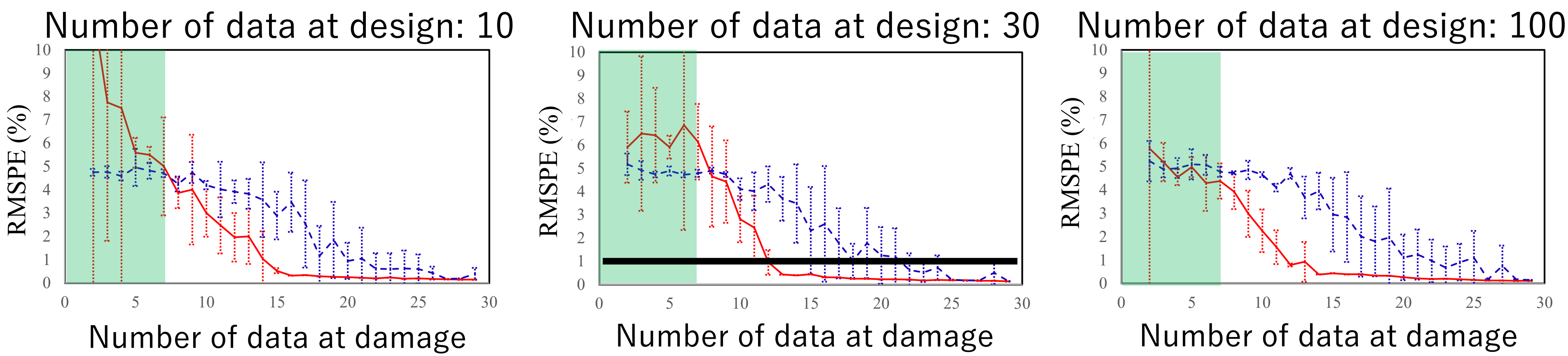
【Result】Accuracy of TL-GPRSM

Predict Maximum Stress （10 Trials）

— TL-GPRSM

— Without transfer learning

$$\mathrm{RMSPE} = 100\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2}$$

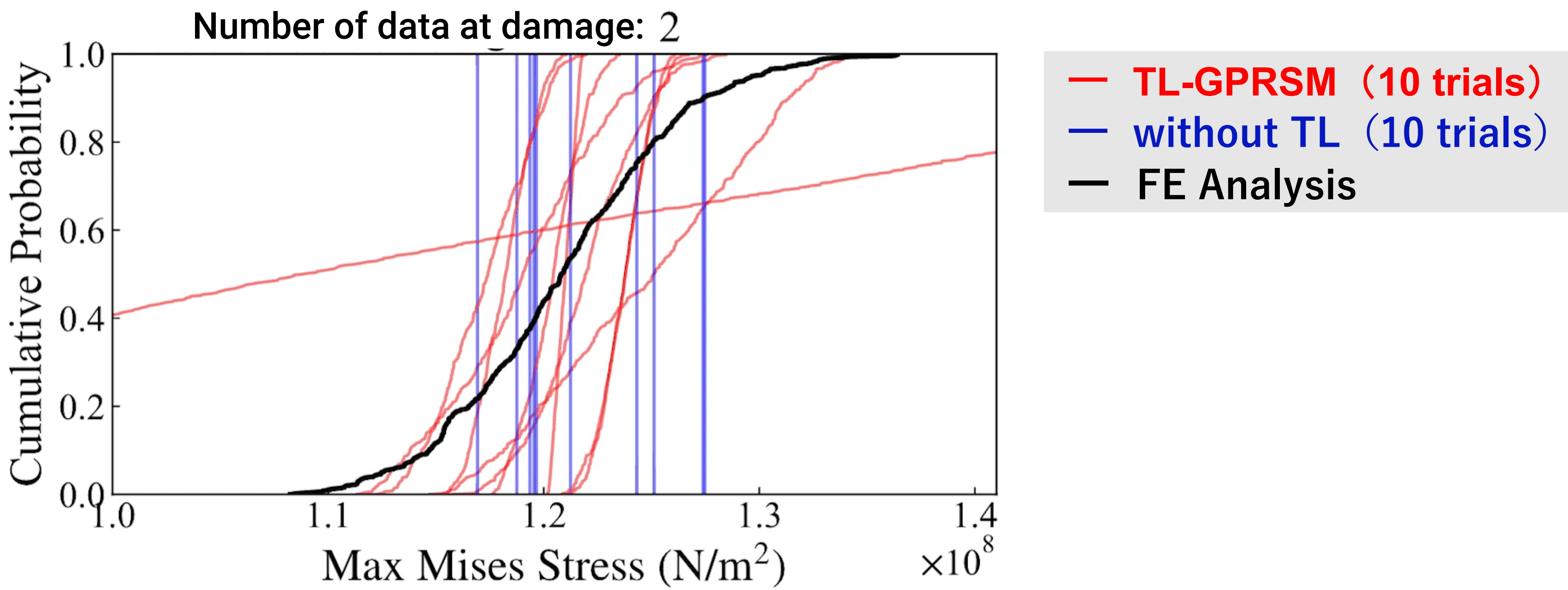


- **TL-GPRSM converges to the number of training data faster, regardless of the number of data at design time, in some cases with 40% less data than the model without transfer learning**
- **The greater the number of data at design time, the higher the accuracy of TL-GPRSM**

【Result】Predicted Distribution of Maximum Stress

Predicted Distribution of Maximum Stress（10 trials）

Number of data at design：30

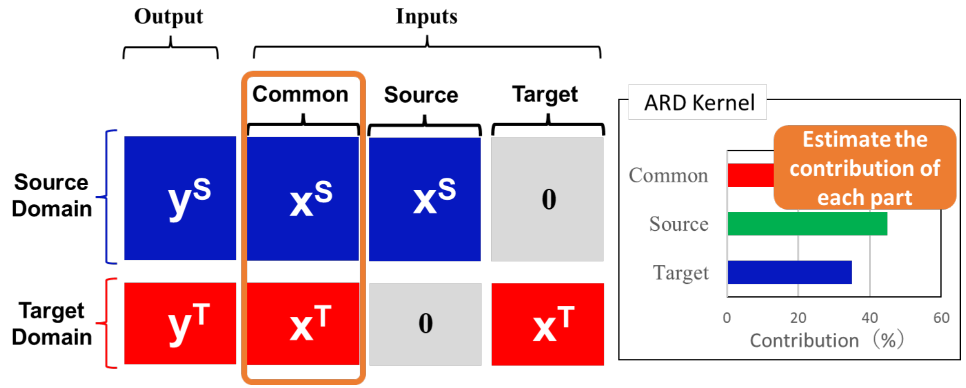


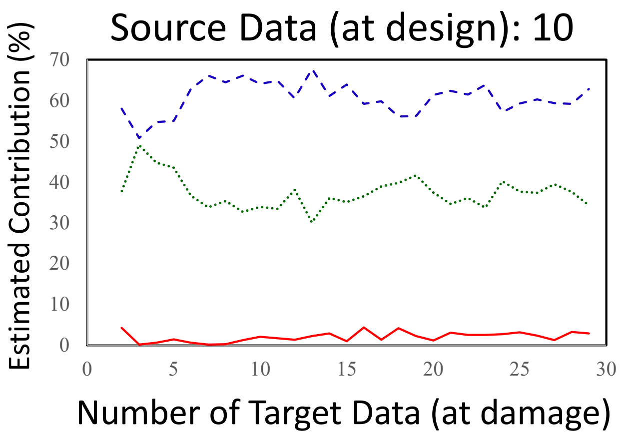
- **TL-GPRSM converged faster on the number of training data than the surrogate model without transfer learning**
- **TL-GPRSM predicted a distribution shape closer to that by FE analysis than the SM without transfer learning for the same number of training data**

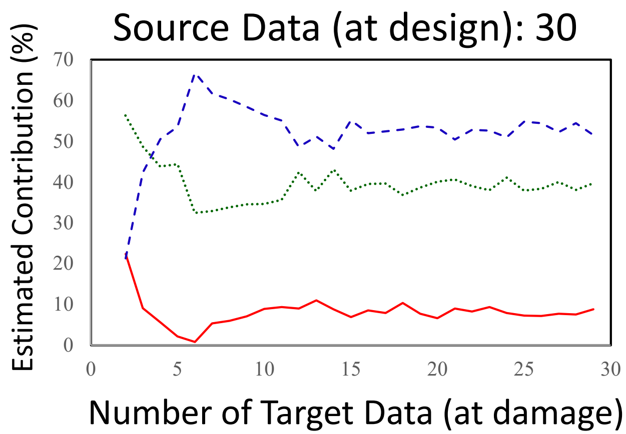
【Result】 parameter contribution estimation by ARD

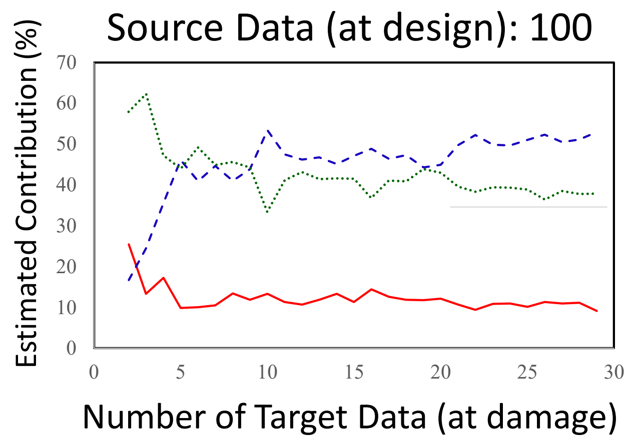
Contribution of each part

— Common part
— Target part
— Source part







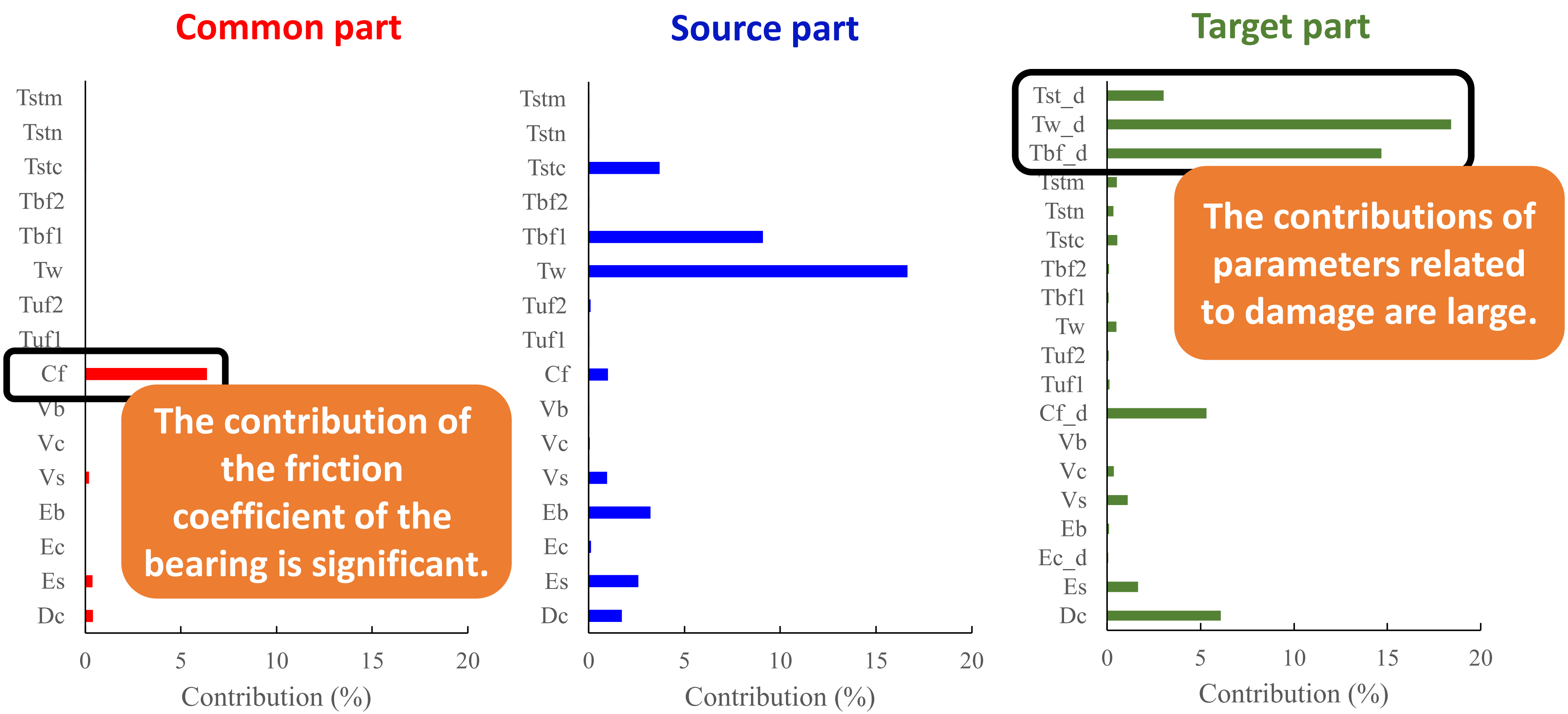


- **The higher the number of source data, the higher the contribution of the common part, and the greater the effect of transfer learning.**
- **The number of source data (30) and the number of source data (100) converged to roughly the same contribution.**

【Result】 parameter contribution estimation by ARD

Contribution of each uncertain parameter

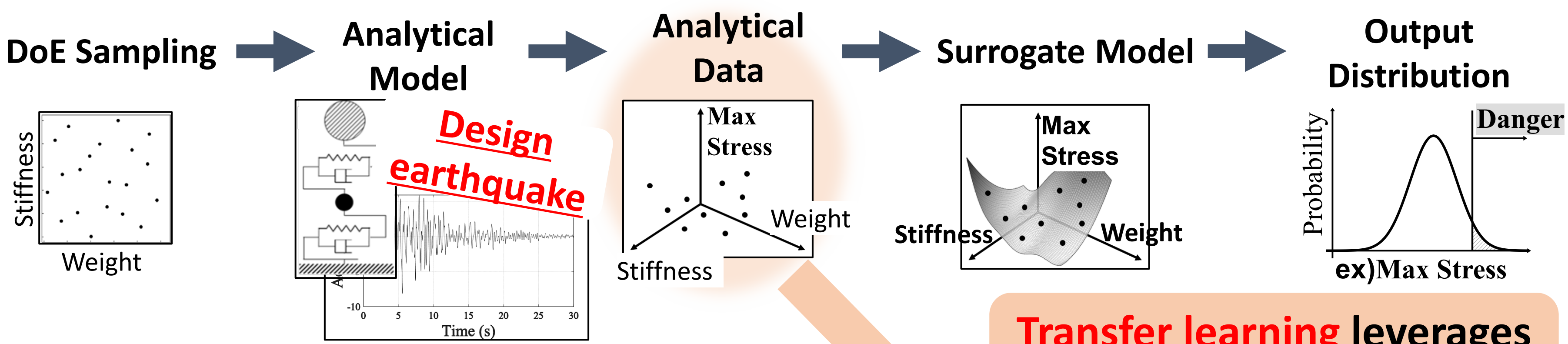
Number of data at design : 30
Number of data at damage : 15



- **ARD is able to properly estimate the contribution.**

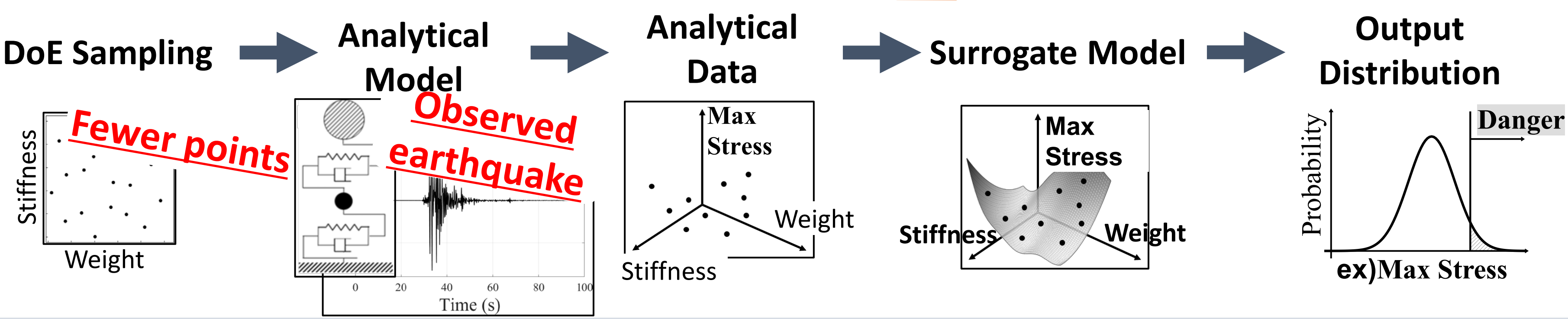
Another story building TL-GPRSM

When designing a bridge



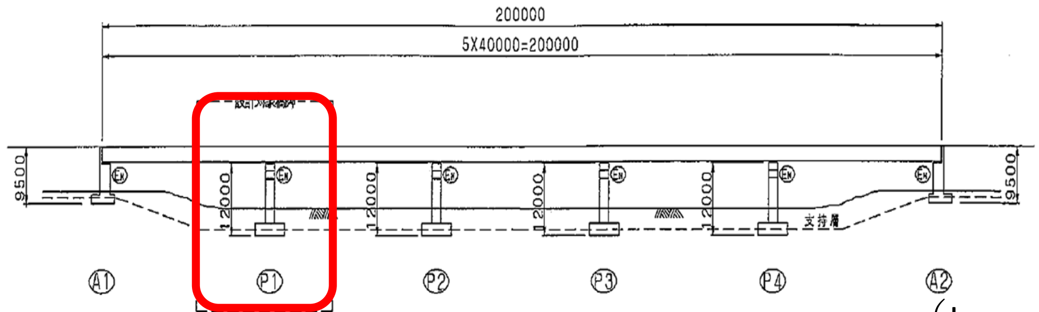
Transfer learning leverages design-time data

When evaluation of existing bridges

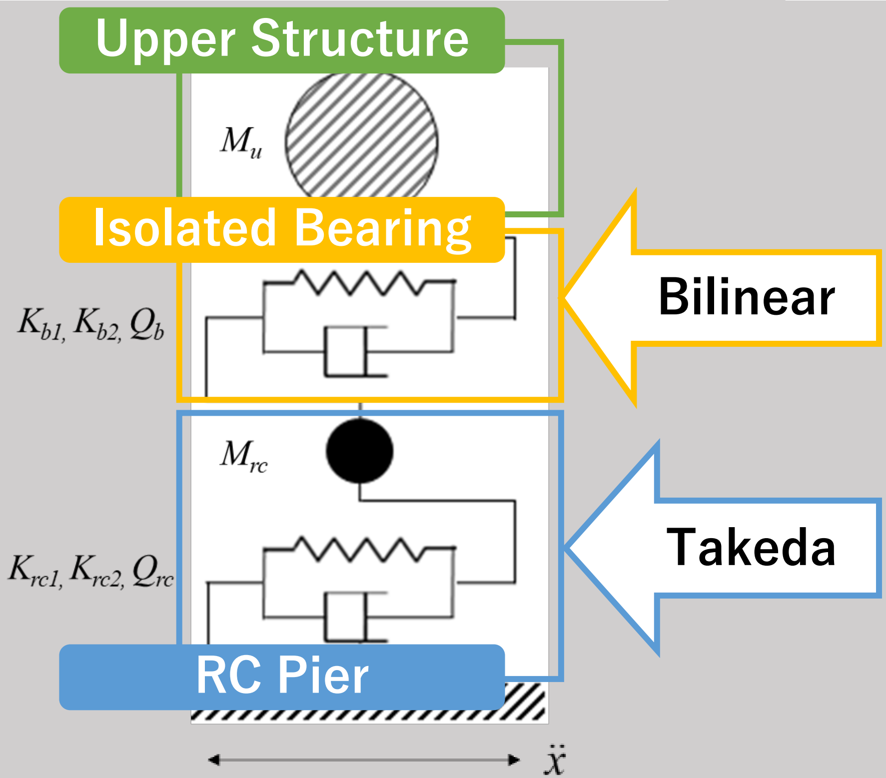


Reduced computational cost for reliability assessment of existing bridges by building a TL-GPRSM using transfer learning

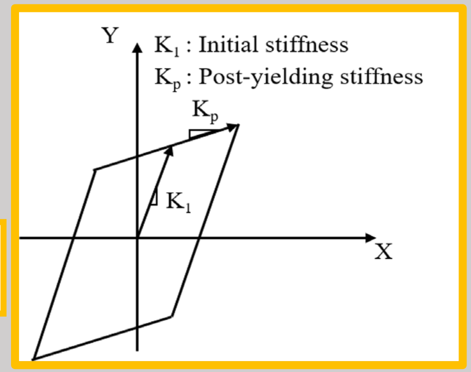
Analytical model of isolated RC piers

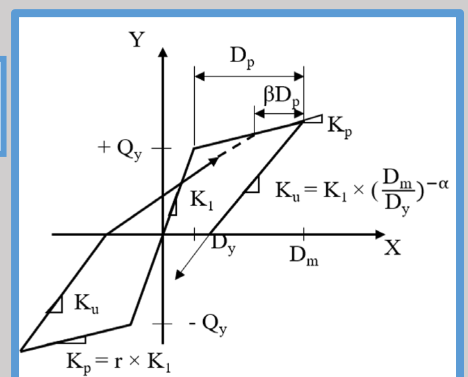


（Japan Road Association, 1997）



2DOF model of Isolated RC Pier





Analysis Parameters

Integration：

- Newmark - β
- Newton Raphson

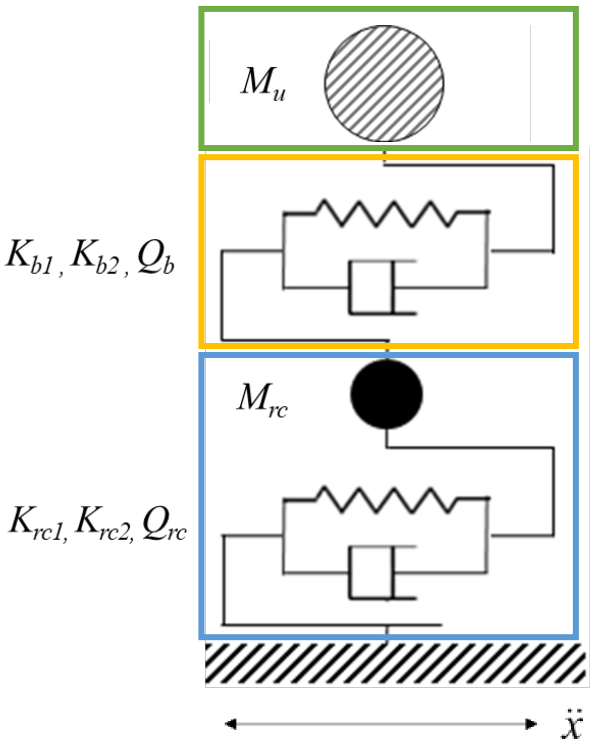
Time Increment： 0.001s

Rayleigh damping

Damping Ratio：

- 2% for pier
- 0% for bearing

Uncertainty parameter setting



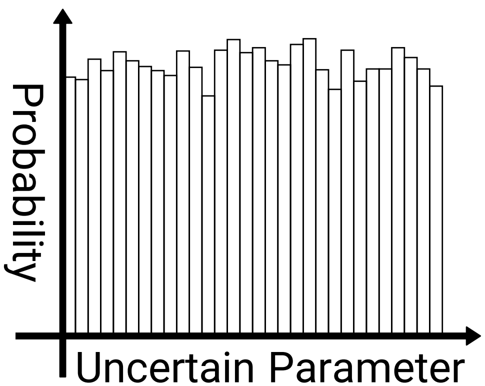
Parameters		Nominal	Uncertainty
Superstructure	weight (Mu)	604000 kg	Uniform Distribution ± 10 %
Seismic Isolation Bearing	Primary stiffness (Kb1)	40023.2 kN/m	
	Secondary stiffness (Kb2)	6154.4 kN/m	
	Yield load (Qb)	1117.2 kN	
RC Pier	weight (Mrc)	346300 kg	
	Primary stiffness (Krc1)	110000 kN/m	
	Secondary stiffness (Krc2)	8250 kN/m	
	Yield load (Qrc)	3399 kN	

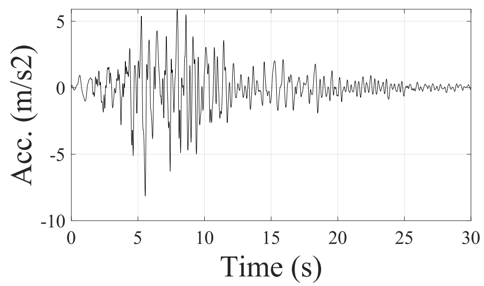
（Reference: Japan Road Association, 1997）

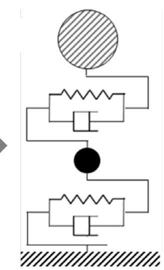
Reliability Analysis Overview and Input/Output

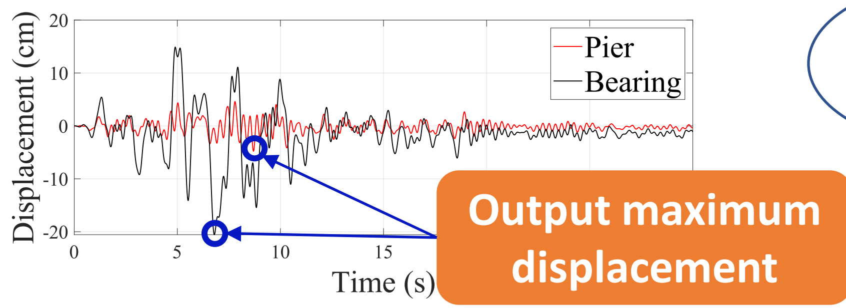
Uncertain Parameters → **Seismic Response Analysis** → **Maximum Displacement (Pier and Bearing)**

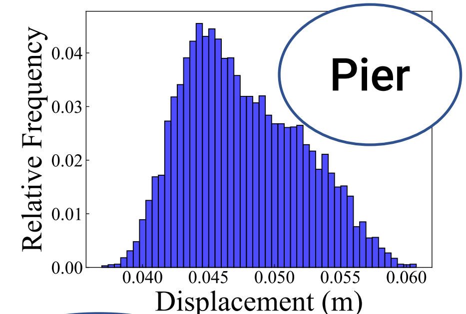
8 variable uncertainty

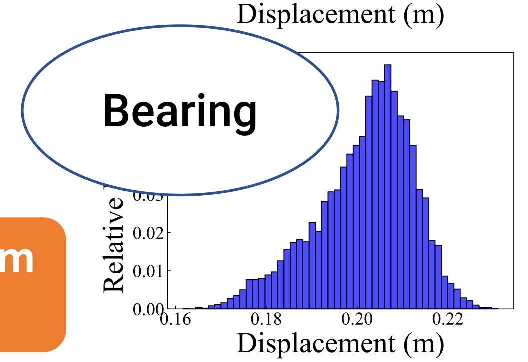












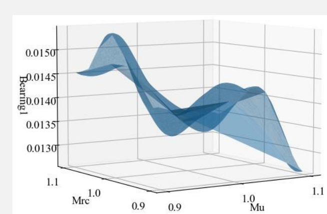
Surrogate model inputs and outputs

Inputs

Uncertainty Parameters

8 Variables

Surrogate Model



Outputs

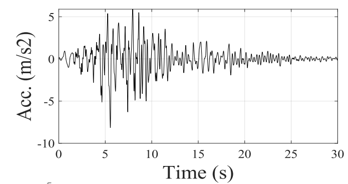
Maximum Displacements

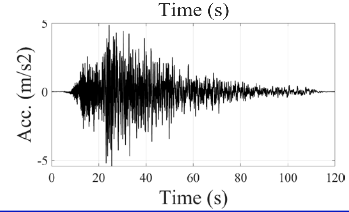
of Pier and Bearing

Datas for Transfer Learning

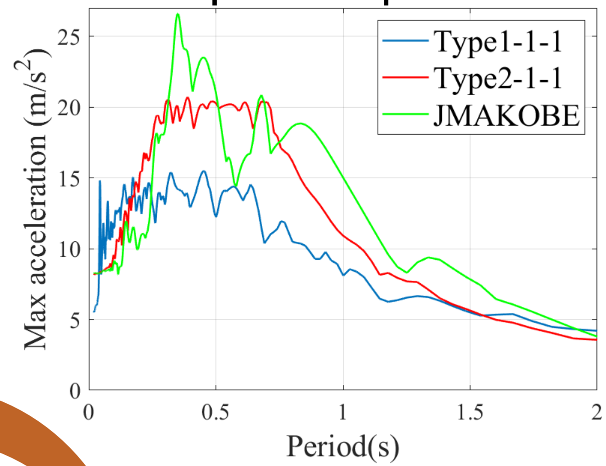
Source Data: **Level2-Type1-1-1 (200 data) and Level2-Type2-1-1 (200 data)**

Maximum Displacement	Uncertain Parameter
y^S	x^S





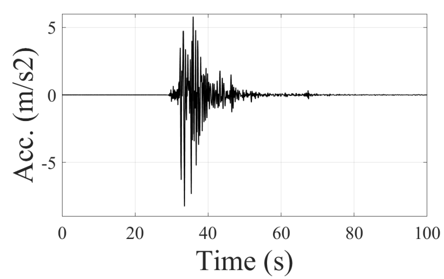
Response Spectrum



Transfer Learning

Target Data: JMA-KOBE Earthquake

Maximum Displacement	Uncertain Parameter
y^T	x^T

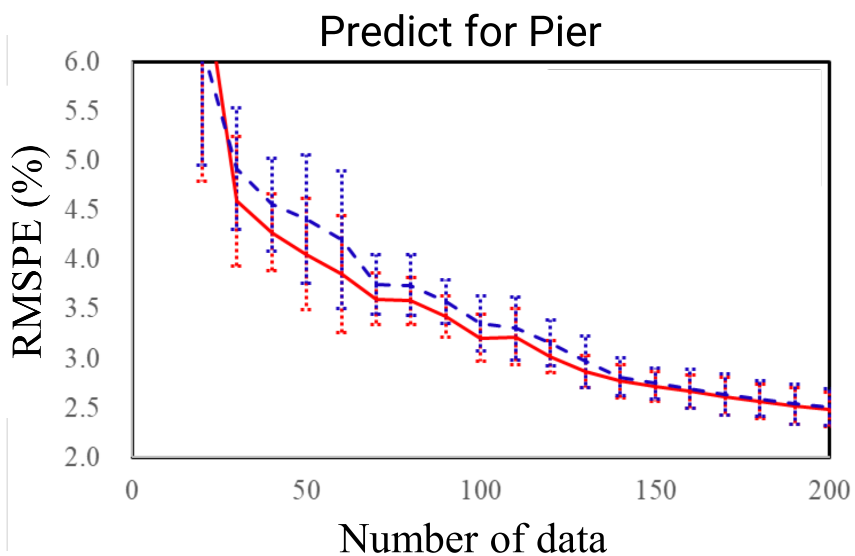


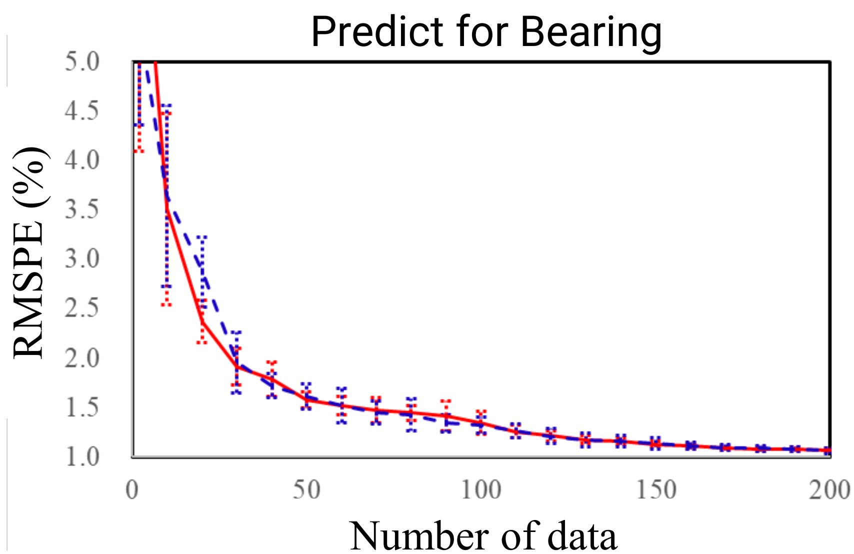
【Result】Accuracy of TL-GPRSM

Predict Maximum Displacement （10 Trials）

— TL-GPRSM
— Without transfer learning

$$\mathrm{RMSPE} = 100\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2}$$



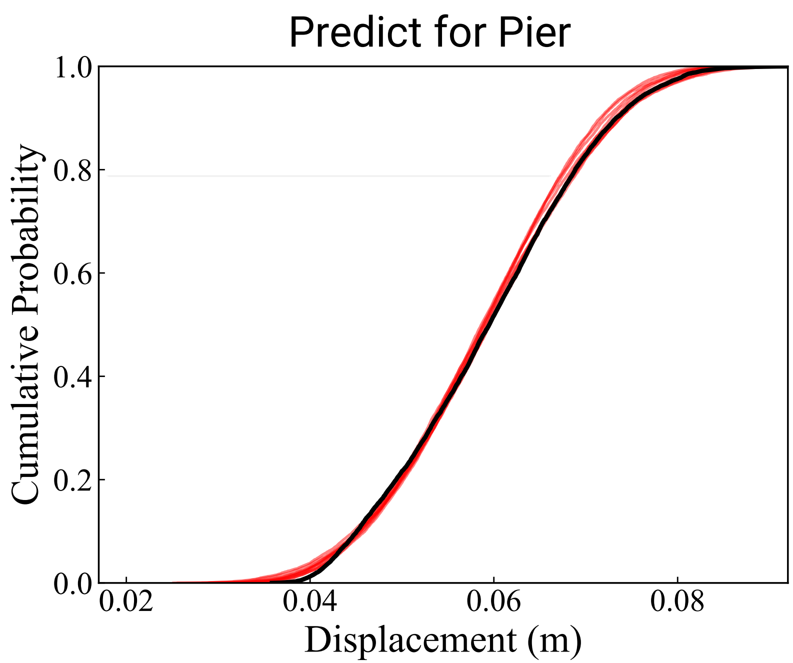


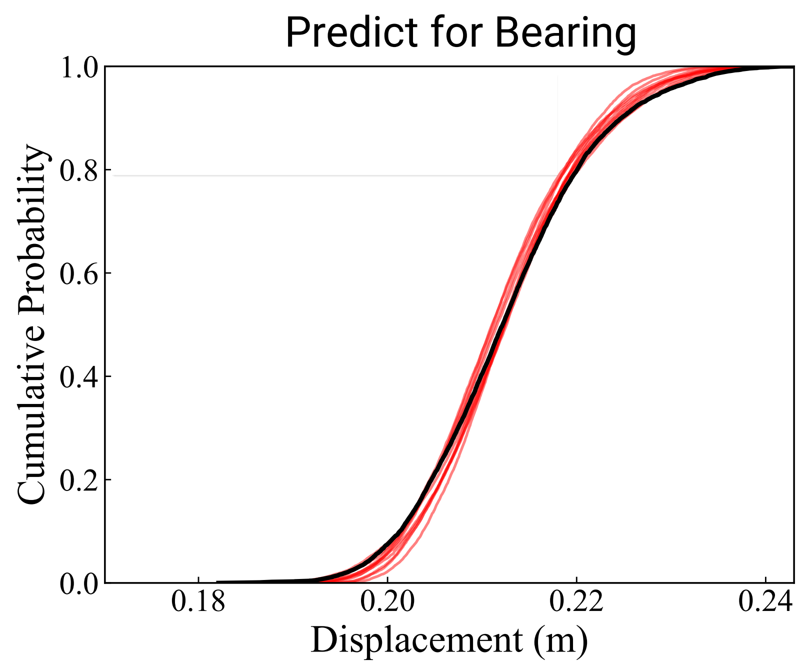
- In predicting the maximum displacement of the Pier, TL-GPRSM was more accurate than the SM without transfer learning
- In the prediction of the bearing, the presence or absence of transfer learning did not affect the prediction accuracy.

【Result】 Predicted Distribution of Maximum Displacement

Predicted Distribution of Maximum Displacement（10 trials）

— TL-GPRSM
— 2DOF model



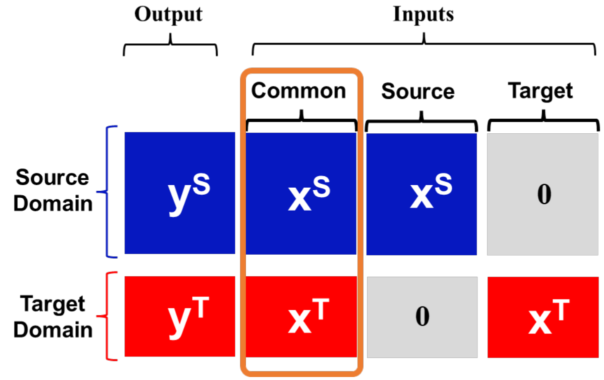


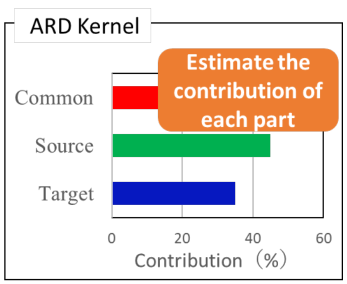
- **For Pier, the maximum displacement distribution was predictable**
- **For Bearing, the TL-GPRSM was able to roughly predict the maximum displacement distribution, but was not able to properly predict the distribution shape at the tail**

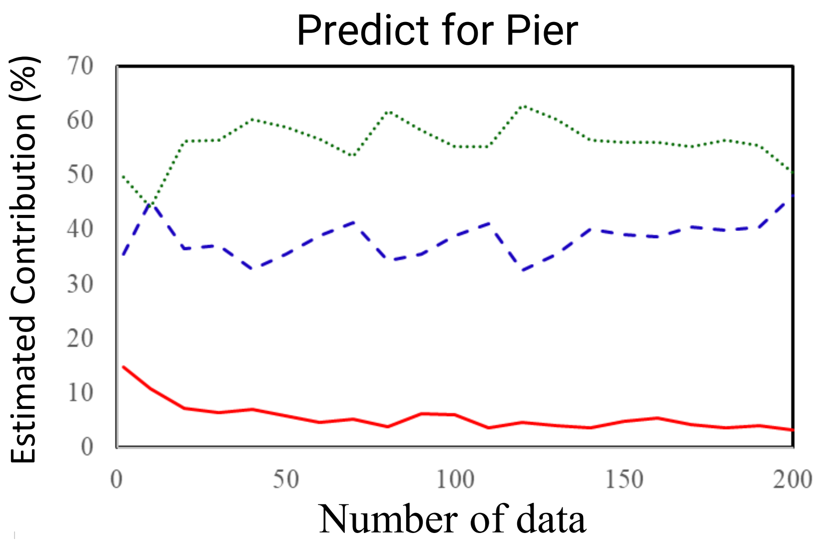
【Result】 parameter contribution estimation by ARD

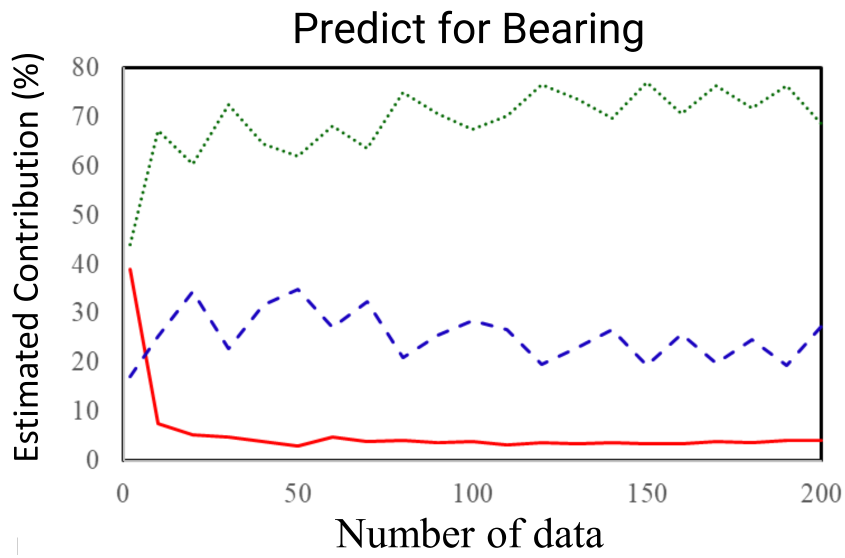
Contribution of each part

- Common part
- Target part
- Source part









- **In general, the contribution of the Common part was smaller than the surrogate model to the analysis in the previous case, converging to about 4% or less.**

Conclusion and Future work

Conclusion

- A transfer learning Gaussian process regression surrogate model (TL-GPRSM) was proposed and applied to evaluate the active load performance of a corrosion-damaged steel plate girder bridge by using design data for post-damage analysis
  - Looking at RMSPE, TL-GPRSM achieved a reduction in computation cost of over 40%
  - The effectiveness of transfer learning was higher the greater the number of source data
- TL-GPRSM was used for seismic response analysis with different input seismic motions, and the data obtained with the seismic design motion was used during the analysis with the observed seismic motion
  - The accuracy of TL-GPRSM was slightly higher than without transfer learning
  - The contribution of the Common part, which measures the effect of transfer learning, was generally lower than in the first case analysis

Future Work

- Combined with adaptive sampling, which preferentially samples points that have a significant impact on the performance of the surrogate model, the computational cost could be further reduced