

# Gaussian process regression surrogate model for dynamic analysis to account for uncertainties in seismic loading

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# Uncertainties in infrastructures

- Infrastructures such as bridges are designed for load and strength.
- However, during the service life, **structures may deteriorate and suffer damage**.
- This is due to the **difference between design and reality**. In reality, there are **many uncertainties**.
- A reliability analysis is required that considers uncertainties related to loads and structural strength.



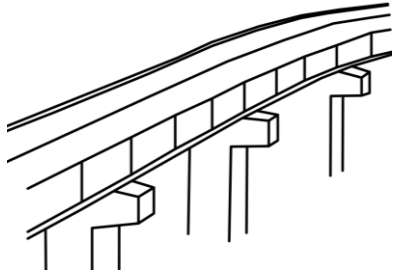
(MLIT, Measures to prevent roads from aging, Aging Status)  
(MLIT, Anti-aging Initiatives)



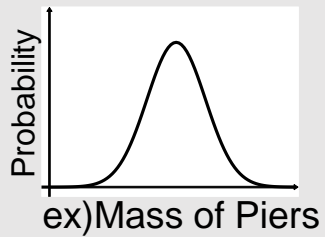
(JSCE, Steel Structure Committee)

# Reliability Analysis Flow

## Target

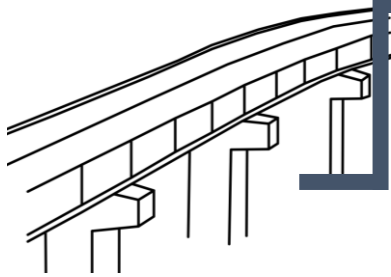


## Considering Uncertainties



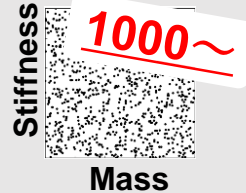
# Reliability Analysis Flow

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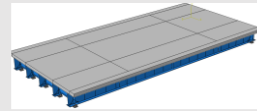


General Reliability Analysis

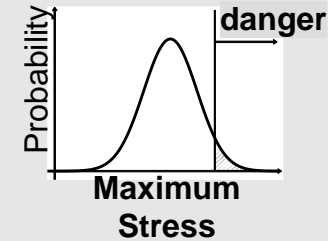
Monte Carlo Sampling



Analytical model

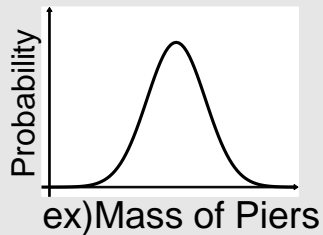


Output distribution

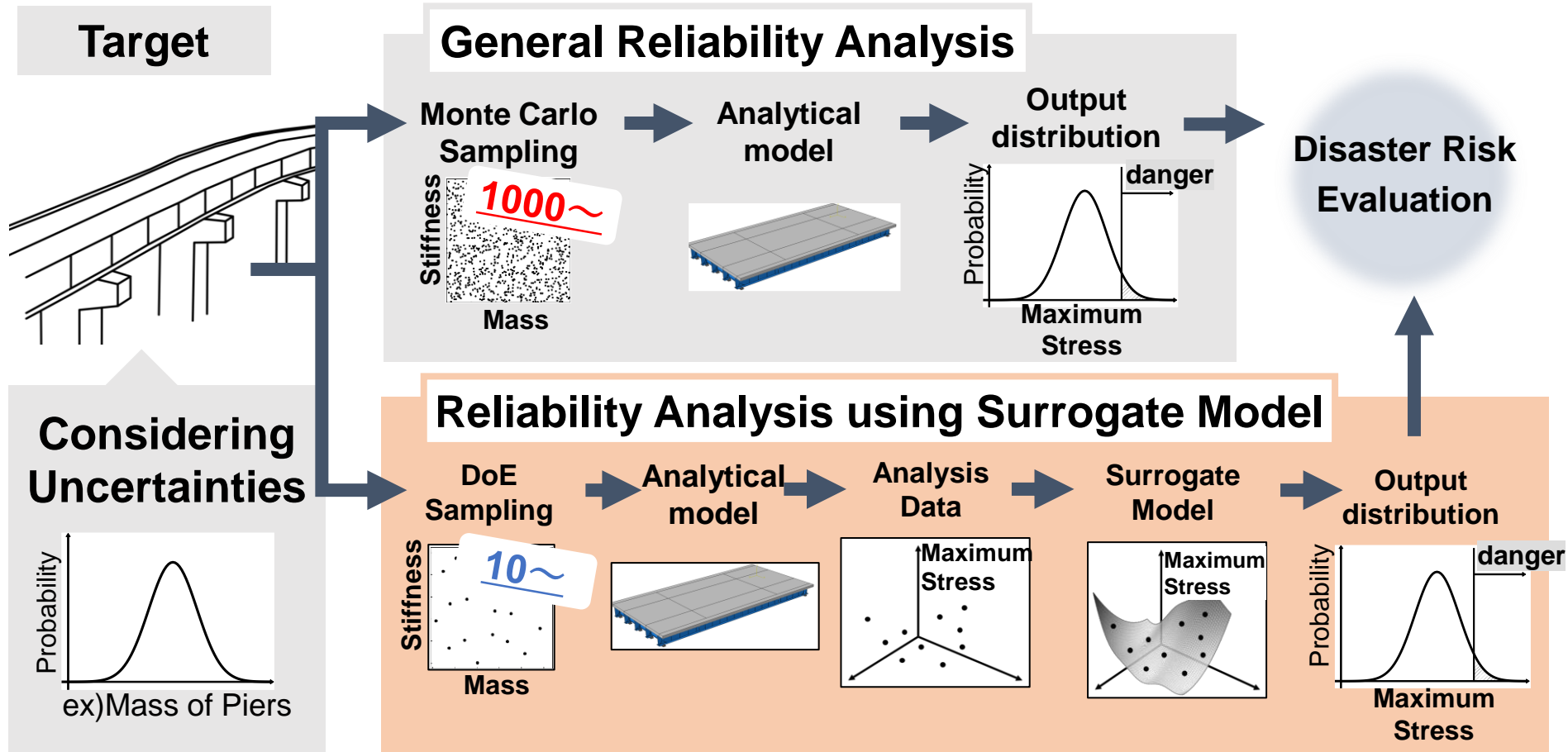


Disaster Risk Evaluation

Considering Uncertainties



# Reliability Analysis Flow



➔ **Surrogate models can reduce computational cost of reliability analysis**

## 【Previous Studies】 Surrogate model for seismic response analysis

### Abbiati et al. 2021

- Using parameters of **artificial ground motions** and structure as inputs
- Constructed surrogate model for seismic risk analysis of piping

*(Journal of Loss Prevention in the Process Industries, Vol.72)*

**Cannot input actual ground motion**

# 【Previous Studies】 Surrogate model for seismic response analysis

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## Zhang et al. 2020

- Seismic waveforms are input using **convolutional neural networks (CNN)**
- Constructed surrogate models for seismic response analysis of buildings

*(Engineering Structures, Vol.206)*

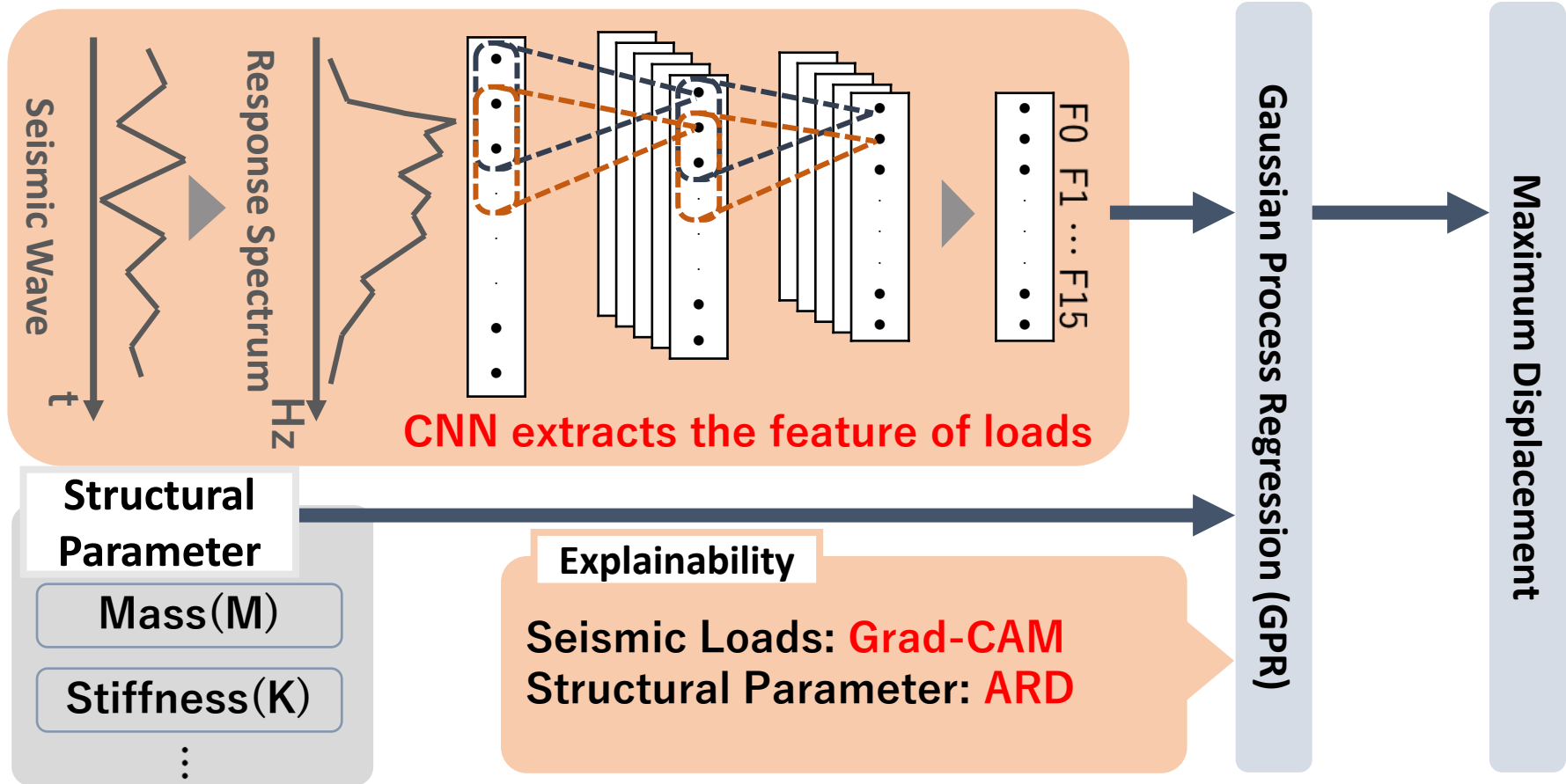
- **Not consider structural uncertainty**
- **Unclear why the result is obtained**

## Issue

- **Considers both actual ground motion and structural parameters**
- **Be able to explain why the predicted results are obtained (**Explainability**)**

# 【Objective】 Deep kernel learning surrogate model

## Feature extraction of seismic loads



Constructing **explainable** deep kernel learning surrogate model with CNN and GPR to reduce computational costs on seismic risk analysis



# Gaussian Process Regression (GPR) with ARD Kernel

## GPR

- Nonparametric
- Non-linear regression

$$y = f(\mathbf{x})$$

$$f \sim GP(\mathbf{0}, k(\mathbf{x}, \mathbf{x}'))$$

$$\mathbf{y} \sim \mathcal{N}(\mathbf{0}, \mathbf{K})$$

$\mathbf{x}$  : input vector

$\mathbf{y}$  : output vector

$k$  : kernel function

$\mathbf{K}$  : kernel matrix

## Kernel Matrix

$$K_{nm} = k(\mathbf{x}_n, \mathbf{x}_m)$$

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## ARD Kernel Function

ARD : Automatic Relevance Determination

Matern5/2 kernel

$$k(\mathbf{r}) = \sigma \left( 1 + \sqrt{5} \sum_{i=1}^D \frac{r_i}{l_i} + \frac{5}{3} \sum_{i=1}^D \frac{r_i^2}{l_i^2} \right) \exp \left( -\sqrt{5} \sum_{i=1}^D \frac{r_i}{l_i} \right)$$

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### Length Scale ( $l_i$ )

Represents the contribution of each input variable to the output

## ARD Kernel

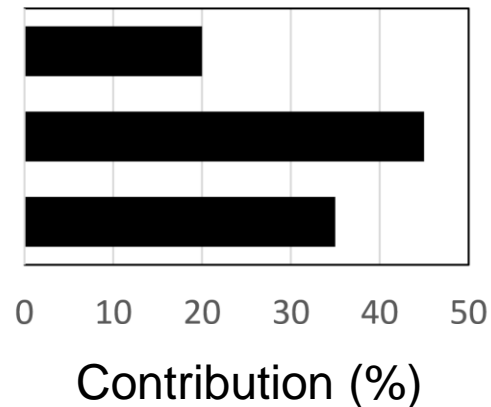
Estimate the contribution of input parameters

Ex)

Poisson's ratio

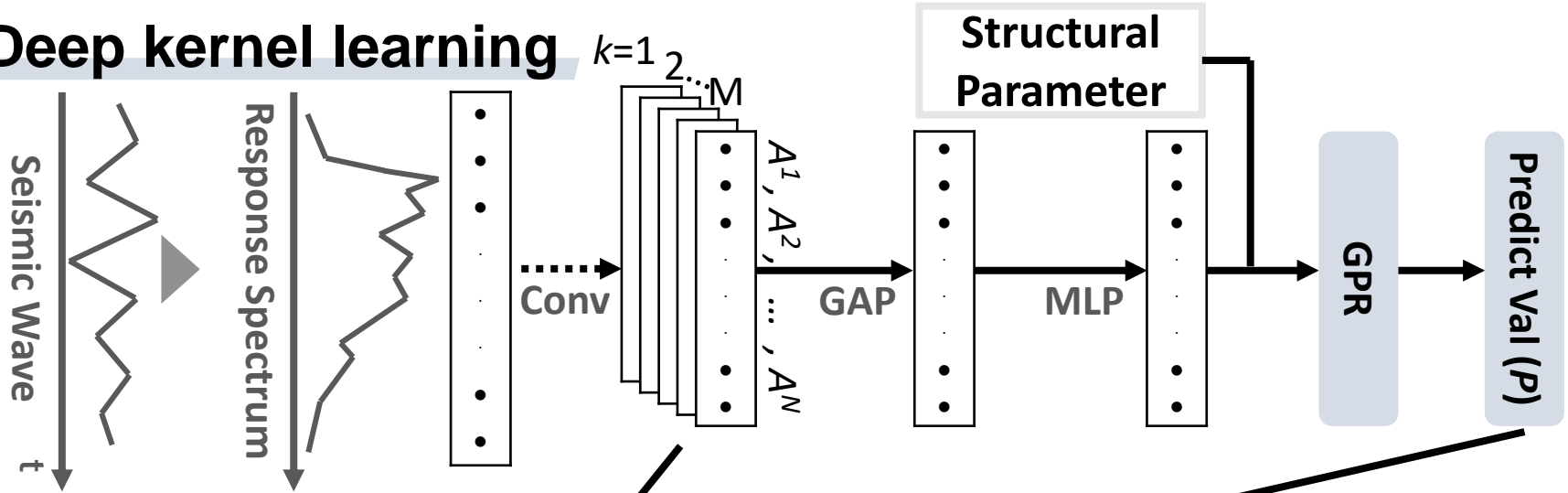
Young's modulus

Thickness



# Grad-CAM for contribution of seismic loads

## Deep kernel learning

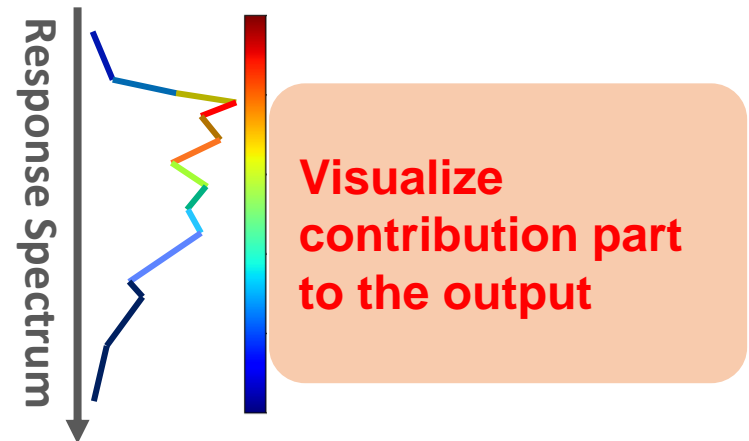


## Grad-CAM

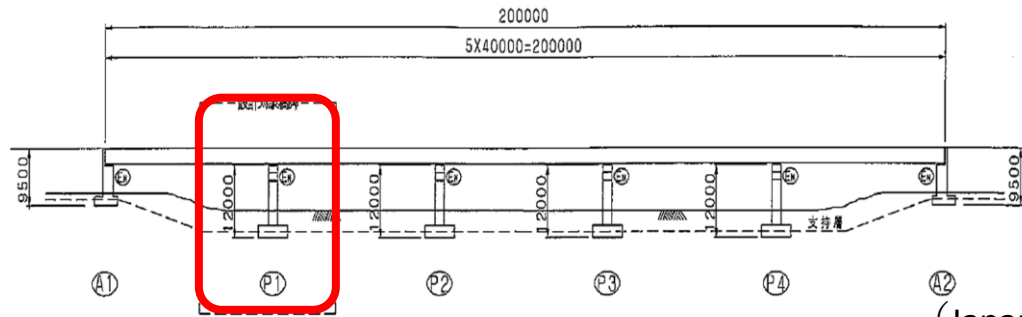
$$\alpha_k = \frac{1}{N} \sum_i^N \frac{\partial P}{\partial A_i^k}$$

$$L_{\text{Grad-CAM}} = \text{ReLU} \left( \sum_k^M \alpha_k A^k \right)$$

Mapping



# Analytical model of an isolated RC pier



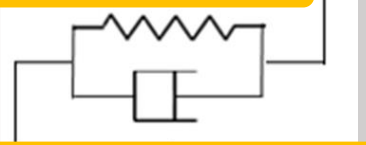
(Japan Road Association, 1997)

## Upper Structure

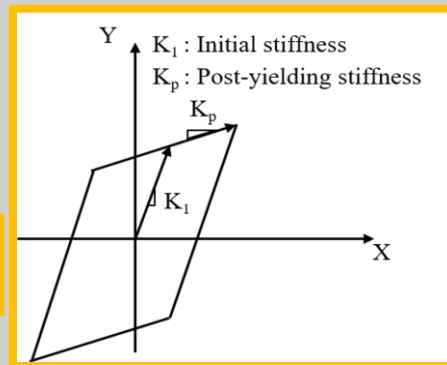


## Isolated Bearing

$K_{b1}, K_{b2}, Q_b$



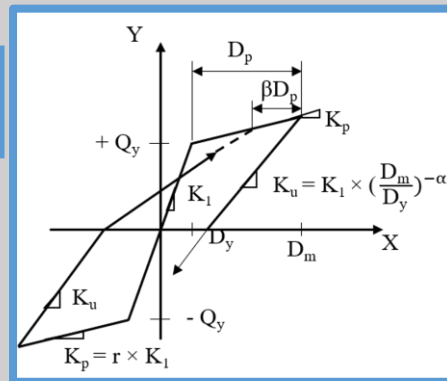
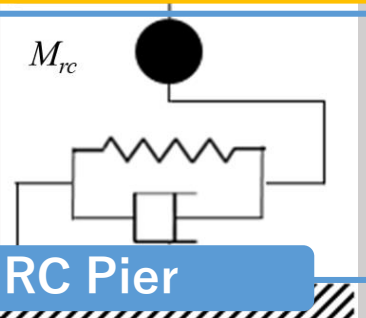
## Bilinear Model



$M_{rc}$

## Takeda Model

$K_{rc1}, K_{rc2}, Q_{rc}$



## RC Pier

$\ddot{x}$   
 2DOF model of Isolated RC Pier

## Analysis Parameters

### Integration :

- Newmark -  $\beta$
- Newton Raphson

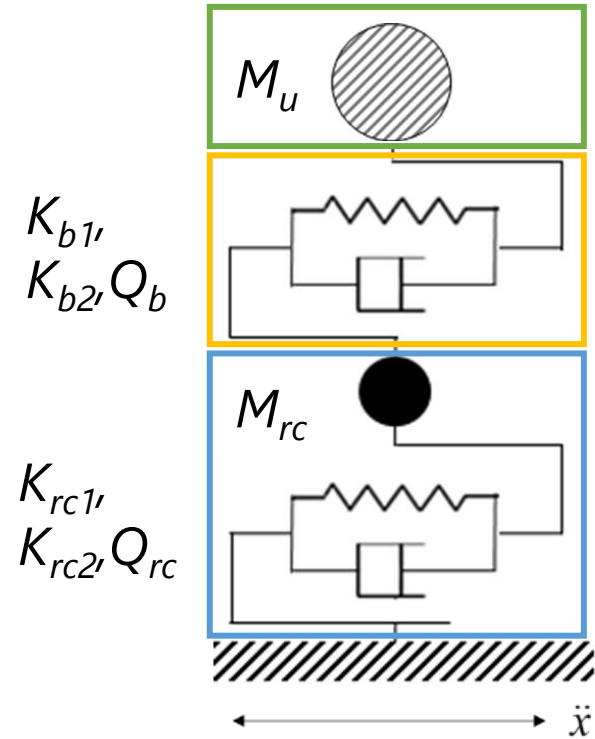
Time Increment : 0.001s

### Rayleigh damping

### Damping Ratio :

- 2% for pier
- 0% for bearing

# Uncertainty parameter setting



Parameters		Nominal	Uncertainty
Superstructure	Weight ( $M_u$ )	604,000 kg	Uniform Distribution $\pm 10\%$
Seismic Isolation Bearing	Primary stiffness ( $K_{b1}$ )	40,023.2 kN/m	
	Secondary stiffness ( $K_{b2}$ )	6,154.4 kN/m	
	Yield load ( $Q_b$ )	1,117.2 kN	
RC Pier	Weight ( $M_{rc}$ )	346,300 kg	
	Primary stiffness ( $K_{rc1}$ )	110,000 kN/m	
	Secondary stiffness ( $K_{rc2}$ )	8,250 kN/m	
	Yield load ( $Q_{rc}$ )	3,399 kN	

(Reference: Japan Road Association, 1997)

# Reliability Analysis Overview and Input/Output

Uncertain Parameters

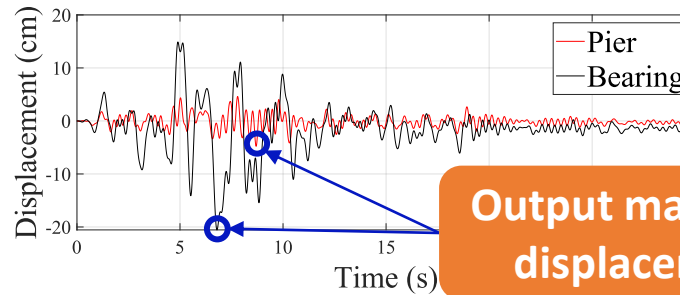
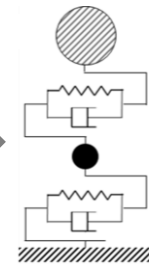
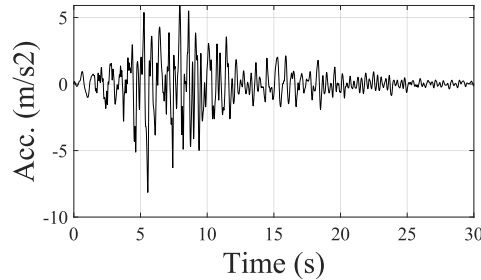
Seismic Response Analysis

Maximum Displacement (Pier and Bearing)

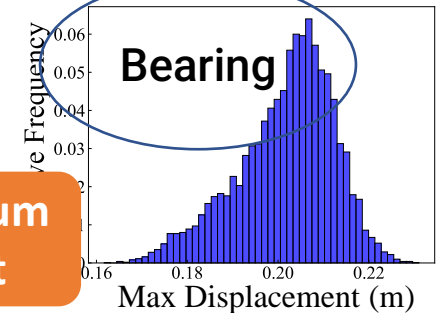
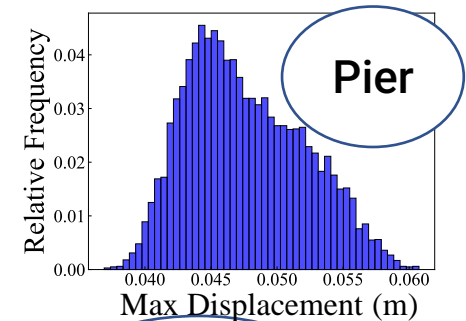
8 structural parameters

20 waves uncertainty

- *JMA KOBE (1996)*
- *Tsurui-nishi (2003)*
- *Tsurui-higashi (2003)*
- *Taiki (2003)*
- *Tokamachi (2004)*
- *Ojiya (2004)*
- *Nagaoka (2004)*
- *Ichinoseki-nishi(2008)*
- *Otsu (2016)*
- *Oguni (2016)*



Output maximum displacement

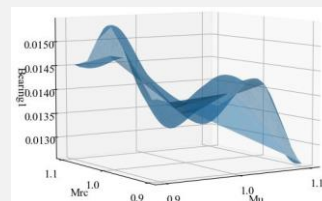


Surrogate model inputs and outputs

Inputs

Structural Parameter  
Response Spectrum

Surrogate Model



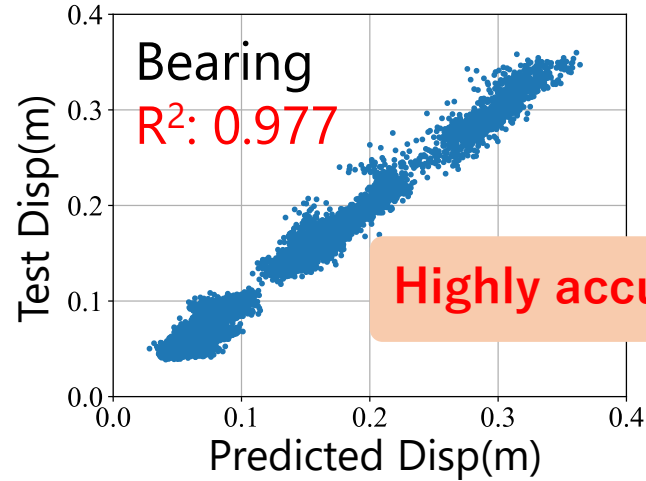
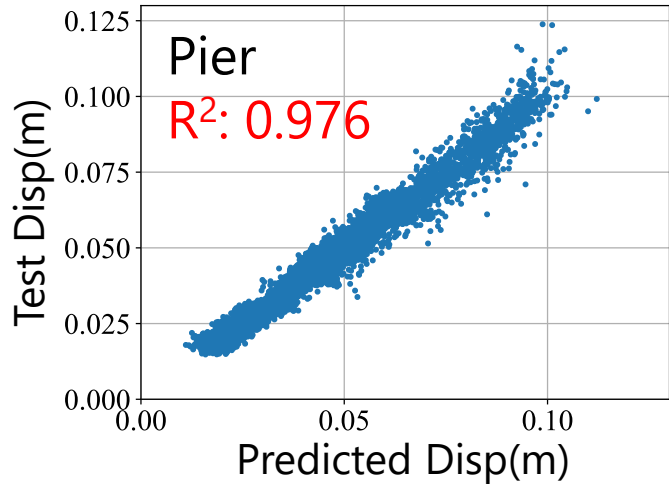
Outputs

Maximum Displacements  
of Pier and Bearing

# 【Result】 Predict Maximum Displacement

## Predicts by surrogate model

Train data : 300 Test data(from analysis) : 10000

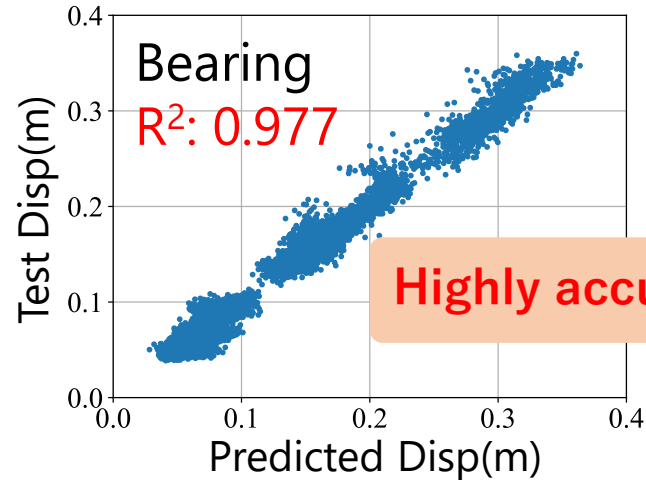
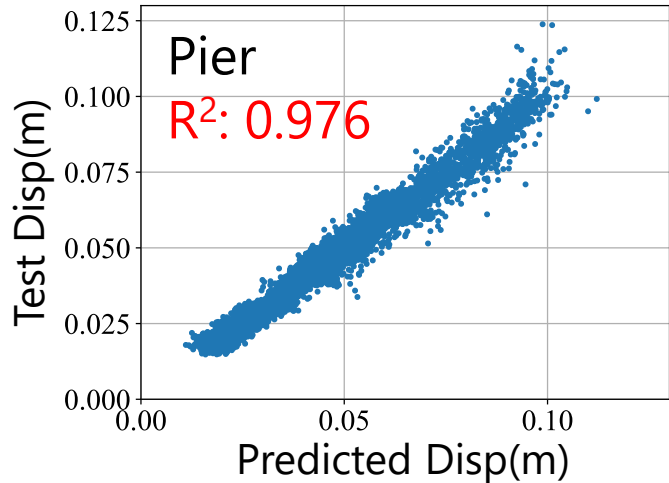




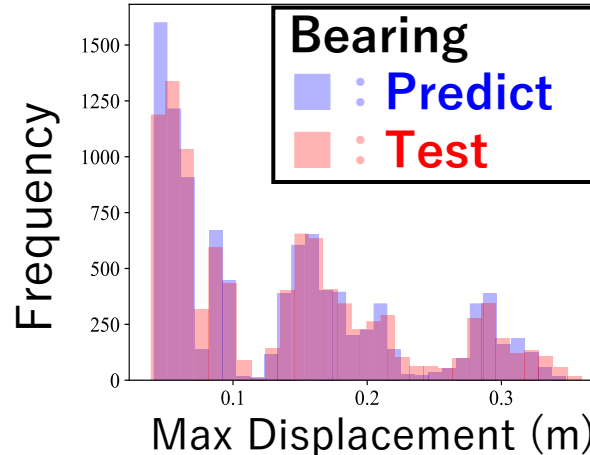
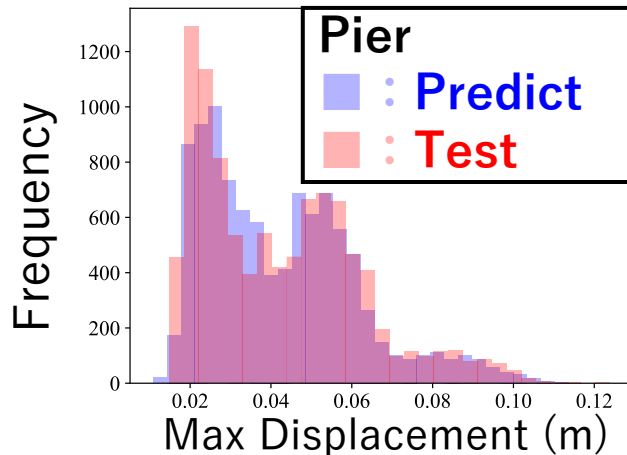
# 【Result】 Predict Maximum Displacement

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Highly accurate predictions.



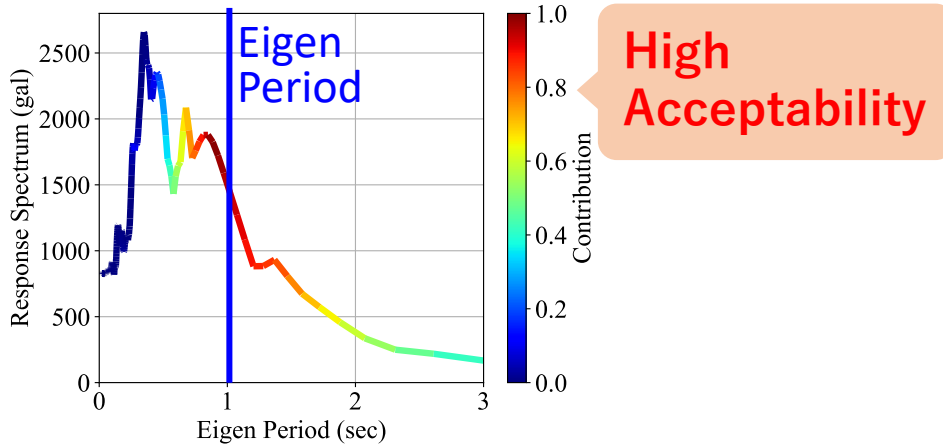
Max displacement distribution is predictable.

Surrogate models can predict with high accuracies

# 【Result】 Estimated Contribution

## Estimated Contribution to Pier's Max Disp

Train num : 300 Test (from analysis) num : 10000

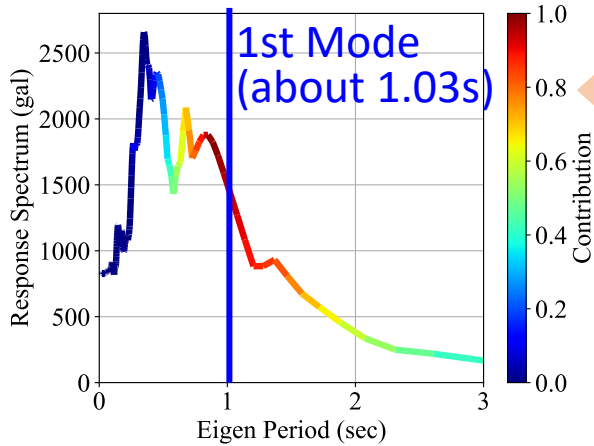


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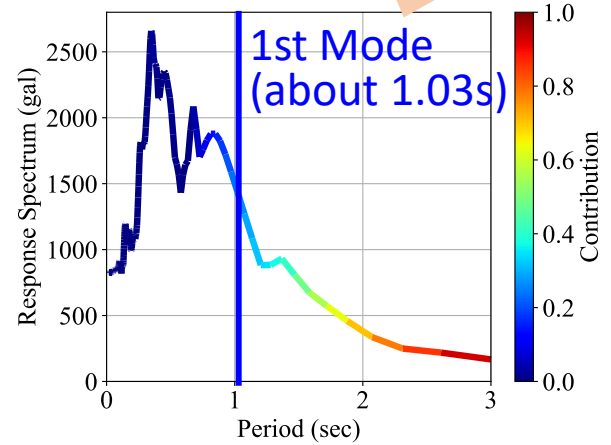
## Estimated Contribution to Pier's Max Disp

Train num : 300 Test (from analysis) num : 10000

Low  
Acceptability



High  
Acceptability

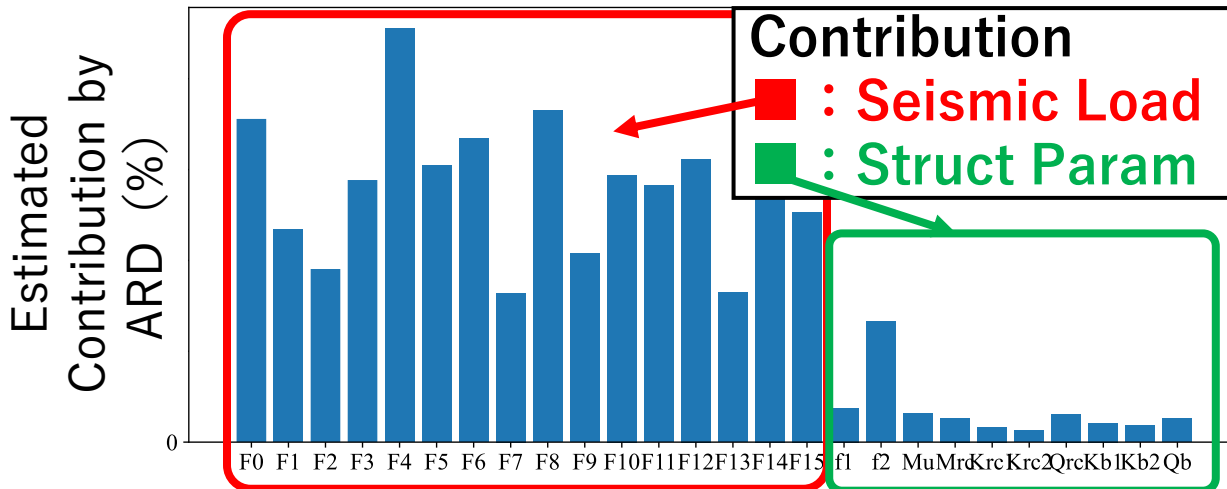
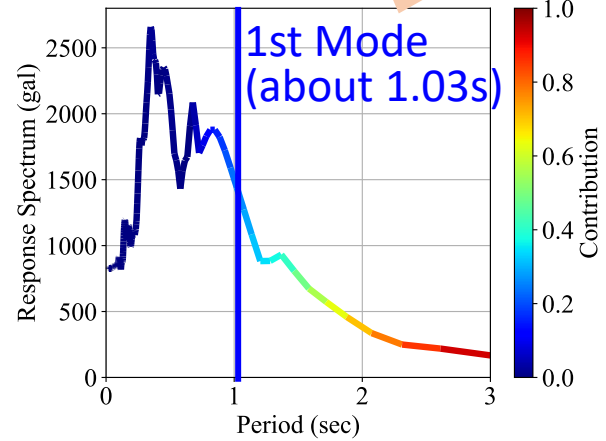
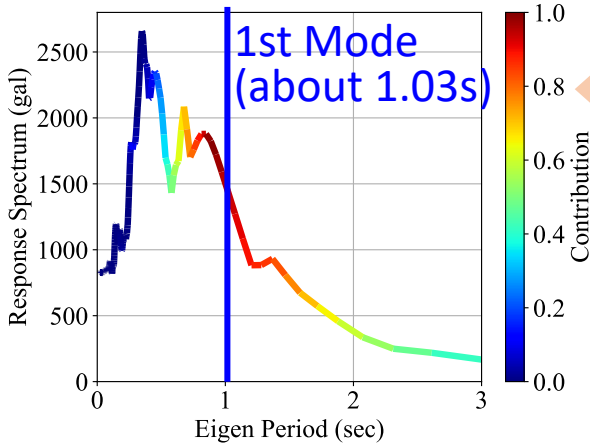


# 【Result】 Estimated Contribution

## Estimated Contribution to Pier's Max Disp

Train num : 300 Test (from analysis) num : 10000

Low Acceptability



Large contribution from seismic loads

Contributions of seismic loads and structural parameters can be estimated

# Conclusion and Future Works

## Conclusion

- A surrogate model for seismic response analysis using deep kernel learning was constructed.
- Seismic load features were extracted by CNN.
- The contributions of seismic loads were estimated by Grad-CAM and structural parameters by ARD.
- The constructed surrogate model **exceeded 0.97 in the R2 index**, and the predicted **distribution was qualitatively consistent with the test data**.
- In some cases, the **Grad-CAM showed a larger contribution close to the natural period**, while in other cases it did not.
- The **ARD-estimated contributions were in agreement with the engineering findings** as well, with the external forces having a larger contribution.

## Future Work

- Combined with adaptive sampling, which preferentially samples points that have a significant impact on the performance of the surrogate model, the computational cost could be further reduced

Thank you for listening.

## Acknowledgement

This study was supported by the JST FOREST Program, Japan

